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Dynamic Informative Advertising of New Experience Goods

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ABSTRACT

This paper analyzes the optimal advertising and price policies of a monopolist who sells a new experience good over time to a population of heterogeneous, forward-looking buyers. We consider informative advertising that can complement or substitute for learning-by-purchasing, and we show that the advertising intensity always peaks during the early stages when the price extracts surplus from the buyers, who are yet to learn their valuation for the good. We also show that even though informative advertising may temporarily raise prices and slow down the learning process, an advertising ban can reduce welfare.

Keywords: monopoly, advertising, experience goods, learning, private information

1. INTRODUCTION

Notwithstanding consumer skepticism of advertising claims for experience attributes emphasized by Nelson (1974), information content of advertisements for frequently purchased experience goods often goes beyond product existence and price (Franke, Huhmann, and Mothersbaugh 2004).¹ In addition to its potential signaling “money-burning” role, depending on the content and medium, advertising can complement or even substitute for learning by purchasing and trying the product. For consumer goods and services—such as a new brand of yogurt, pet food, detergent, or medication; new automobile or homeowner’s insurance; or a new fitness class—information about experience attributes obtained from a product trial may be noisy due to interaction with other products, variability in product attributes, or external factors (Erdem and Keane 1996; Akerberg 2003). In such cases advertising information may complement learning-by-purchasing by raising awareness, focusing consumers’ attention on certain aspects of the product, and helping consumers better evaluate their product trial experience (Kempf and Lacznik 2001). For example, advertisements for prescription and over-the-counter drugs often describe not only the benefits and likely users but also the most common or severe side effects (Appleby 2008). Also, quotations from reviews of a book series or a new cable TV channel, or previews of a movie series, usually provide some indication of not only the product’s overall quality but also of how well it will match the reader’s or viewer’s tastes. Another example of advertising that substitutes for learning-by-purchasing is free product sampling, which may obviate the need for consumers to buy the product to see if they like it (York and Zmuda 2008).²

It is unclear, however, whether firms always have incentives to provide detailed product information and facilitate consumer learning in such ways. In this paper, we study a dynamic model of informative advertising for a new nondurable experience good, asking the following questions: When does the seller offer advertising that complements or substitutes for learning-by-purchasing? How does advertising intensity vary over time? Are consumers better informed as a result of advertising? How does advertising affect prices? What about its effect on consumer and overall welfare?

Our model is an extension of the Bergemann and Valimaki (2006) model (BV model) of monopoly pricing of a new experience good with independent private valuations and consumer learning over time. In the BV model, perfectly informative signals about the individual match value arrive at random, and consumers, in aggregate, gradually learn their valuations by purchasing and consuming the product. Here we assume that the seller can, at a cost, generate additional informative signals that may arrive at different frequencies to consumers who are purchasing the product and ones who are not.

In the BV model, at each point in time there are two segments of buyers: those who have already learned their valuation and those who are yet to do so. The seller’s problem is to choose the pricing and advertising policies that will maximize the discounted sum of profits while managing the learning process with forward-looking buyers. In the BV model the dynamics of the learning process is qualitatively different in a so called mass and niche market. In a mass market, learning-by-purchasing never stops, whereas in a niche market it terminates after the period of low introductory prices.

The learning process that is aided by advertising also evolves differently depending on whether the market is a mass or a niche market and whether advertising exposures complement or substitute for learning-by-purchasing. We find that advertising that complements learning-by-purchasing is offered during the early stages of the aggregate learning process. However, advertising that substitutes for learning-by-purchasing is not offered at all in mass markets but is long-lasting in niche markets. During

¹ For example, in their analysis of a sample of print ads from U.S. magazines, the difference in the number of information cues between ads for nondurable experience and search products was statistically insignificant.

² Between these two polar cases are interactive virtual experience delivered by the Internet that can help transform some experience attributes of a product into search attributes (Daugherty, Li, and Biocca 2008) and the use of analogies to familiarize consumers with the unique experience a product offers (Goode, Dahl, and Moreau 2010). Additional means of product sampling include fragrance samples on paper bound into magazines and taste samples of food and drink products (Vranica 2008; Gerlich, Browning, and Westermann 2010).

the early stages the advertising intensity can exhibit a variety of patterns depending on the parameters of the model such as the degree of complementarity between learning-by-purchasing and advertising, the distribution of actual valuations, production and advertising costs, the discount rate, and the rate of learning from consumption experience unaided by advertising.

Following the marketing literature, we distinguish between the marginal effects of advertising on the current (“immediate”) and future (“long-run”) profits. The immediate effect of advertising is positive when it increases the current demand from uninformed consumers, who favor experiential learning facilitated by advertising. On the other hand, if both purchasing and nonpurchasing buyers are equally likely to learn from advertising, uninformed buyers are not willing to pay for advertising that delivers information regardless of the purchasing decision, and thus the immediate effect is null.

As advertising influences the aggregate learning process, it also has a long-run effect, which can be positive or negative depending on whether the current price is above or below the long-term price. If uninformed buyers are relatively likely to become future customers, and if they constitute the more profitable segment of the market, advertising decreases future profits by changing the composition of demand in a way that disadvantages the seller. However, if uninformed buyers are relatively unlikely to become future customers, and if they require low introductory prices to be willing to experiment with the product, advertising increases future profits by expanding the long-term customer base.

Consider advertising that complements learning-by-purchasing. In a mass market with a small discount rate and a small net average consumer valuation, the advertising intensity falls over time. This happens because the long-run effect is stable while the immediate effect of advertising declines when there are fewer uninformed consumers left in the population. Yet in a niche market, the advertising intensity may at first fall and then rise during the early stages. If advertising substitutes for learning-by-purchasing, as in the case of free product samples, the advertising intensity follows a bell-shaped curve that hits its highest point before learning-by-purchasing terminates. In this case, the choice of advertising intensity is driven by the effect of advertising on future profits, which peaks when the learning process has advanced but is not yet complete.

To evaluate the effects of advertising (or a ban on advertising), we compare the dynamics of purchasing behavior, learning, and prices in equilibrium in the model with and without advertising. Although the learning process is accelerated by advertising that complements experience, advertising that substitutes for learning-by-purchasing may temporarily slow it down by crowding out learning-by-purchasing. We also find that the effects of advertising on the pricing policy are rather nuanced. During the early stages of the learning process the price is determined by the current willingness to pay of uninformed buyers, which may increase or decrease as advertising policy affects both the prices and the probability of learning one’s actual valuation in the future. In a mass market, advertising may raise prices at first but also accelerate their subsequent decline. In a niche market, the outcome can be quite different. Advertising that substitutes for learning-by-purchasing reduces low introductory prices but also shortens the period of time during which they are offered. Yet advertising that complements learning-by-purchasing may have the opposite effect. In the long run, however, any effects of advertising on the pricing policy vanish as the price converges to the static equilibrium price under complete information.

Finally, let us compare the present value of consumer and social welfare in equilibrium with and without advertising. In our model, advertising benefits uninformed consumers. This happens because they begin to retain some of their expected information rent sooner and/or may learn their preferences without purchasing the product. Although advertising temporarily raises prices and may slow down the learning process, consumer welfare improves because the product is initially new to all consumers. Yet advertising does not necessarily increase the present value of social surpluses. In equilibrium, the monopolist cannot commit to the paths of advertising and prices, and thus future advertising may reduce the private and social value of learning today vis-à-vis the private and social value in equilibrium without advertising. In the long run, however, advertising improves social welfare in a niche market and in a mass market in which trades with consumers who have low valuations for the good are not socially efficient.

The rest of the article is organized as follows. After a review of the related literature, we present our model. In Section 3, we set up consumers’ and the monopolist’s optimization problems and derive

optimality conditions. In Section 4, we identify conditions under which the seller advertises in equilibrium based on the characterization of the stopping point of the advertising policy. In Section 5, we discuss the determinants of the dynamic advertising policy. In Section 6, we consider advertising that provides little information for nonpurchasing buyers. In Section 7, we consider advertising that provides information to nonpurchasing buyers. In Section 8, we study the effect of advertising on learning and prices, and welfare economics in Section 9. Section 10 concludes. All proofs are collected in the Appendix.

Related Literature

To our knowledge the current paper is the first to accommodate informative advertising and pricing in a fully dynamic model with forward-looking agents. Our model of advertising contributes to the theoretical literature that analyzes the seller's advertising decision in static settings as well as to the literature that studies the dynamic models of the aggregate response of demand to advertising.

In static settings, the question of whether the monopolist achieves higher profits when consumers are privately more informed about their own valuations for the good has been analyzed by Lewis and Sappington (1994), Johnson and Myatt (2006), Anderson and Renault (2006), and Saak (2008). In Lewis and Sappington (1994) and Johnson and Myatt (2006), the effect of advertising on the aggregate demand is to increase the variance of the distribution of valuations among consumers. If consumers are less informed, they constitute a more uniform group, which makes it easier for the seller to skim consumers. If consumers are more informed, the seller may target consumers with high valuations and raise the price, but information also provides rents to buyers. These authors' main finding is that the seller's profit is typically maximized under either null or full private information.³ In our model, demand dispersion is determined by the share of informed consumers. The seller decides whether and how much to spend on advertising in each period while taking into account the short-term and long-term effects of advertising on the value of information for the uninformed (inexperienced) consumers and the composition of demand.

In dynamic settings, optimal advertising and pricing policies have been extensively studied in the marketing literature.⁴ In an early paper, Nerlove and Arrow (1962) model advertising as an investment in the stock of goodwill that increases current revenues and depreciates over time. Another class of dynamic advertising models builds on the approach of Vidale and Wolfe (1957) that postulates a direct relationship between sales and advertising. In the Vidale–Wolfe model, current sales increase due to advertising, which affects only the unsold portion of the market, and current sales decrease due to forgetting in the sold portion of the market. An important feature of their model is that the finiteness of the size of the market (the saturation level), rather than the concavity of the seller's revenue function, leads to diminishing returns on advertising. The response of demand to informative advertising in our model shares features of both the Nerlove–Arrow and the Vidale–Wolfe models. Informative advertising can be seen as an investment in the stock of goodwill, provided that dealing with more informed consumers benefits the seller (otherwise, advertising would generate a stock of “ill will”). Informative advertising in our model is also subject to diminishing returns because it evokes a response only from the uninformed segment of the market.

This paper also complements more recent investigations of the optimal product sampling effort over time. Heiman et al. (2001) analyze an extension of the Nerlove–Arrow model in which free samples have both the immediate sales and the long-run goodwill-building effects. Bawa and Shoemaker (2004)

³ Anderson and Renault (2006) and Saak (2008) show that the seller typically prefers to convey only partial product information rather than null or full when the set of private information structures is larger.

⁴ For example, optimal dynamic advertising policies in diffusion models of the aggregate sales response to advertising in monopoly are studied in Dockner and Jorgensen (1988); Mesak and Zhang (2001); and Sethi, Prasad, and He (2008). Chintagunta, Vithala, and Vilcassim (1993) study dynamic pricing and advertising policies in spatial duopoly for nondurable experience goods. In their model, consumers are uncertain about horizontal attributes, and their beliefs are determined by the consumption experience that evolves according to advertising effects of the types described by Nerlove and Arrow (1962) and Vidale and Wolfe (1957).

distinguish among three effects of free samples on sales: (1) an acceleration effect, whereby consumers begin repeat purchasing of the sampled brand earlier than they otherwise would; (2) a cannibalization effect, which reduces the number of paid trial purchases of the brand; and (3) an expansion effect, which induces purchasing by consumers who would not consider buying the brand without a free sample. Our model enriches these analyses because informative advertising not only interacts with learning but also affects the equilibrium pricing policy.

2. MODEL

We consider an extension of the Bergemann–Valimaki (BV) model of dynamic pricing of new experience goods with the learning rate that is influenced by advertising. The model is formulated in continuous time. At each instant $t \in [0, \infty)$, a seller (monopolist) with constant unit production cost $k \geq 0$ offers a single nonstorable, nonreturnable product to a unit continuum of risk-neutral consumers with unit demands. The monopolist and the buyers discount the future rents at rate $r > 0$.

Every consumer is characterized by his idiosyncratic willingness to pay for the product, θ . The good is an experience good, and the true value of θ is initially unknown to the buyer and the seller, who also cannot distinguish between informed and uninformed consumers. True valuations, θ , do not change over time, and in the beginning of the game they are independently drawn from the probability distribution function $F(\theta)$ with support $[\theta_l, \theta_h]$ and differentiable density function $f(\theta)$. This distribution is common knowledge. We assume that $(1 - F(\theta))(\theta - k)$ is strictly concave in θ , and we let $v = E[\theta] = \int_{\theta_l}^{\theta_h} \theta dF(\theta)$.

Consumers learn their actual valuation for the product by buying and consuming the product, but learning-by-purchasing can be enhanced or supplanted by exposures to advertising messages from the seller. Specifically, we assume that a perfectly informative signal arrives to each consumer who purchases the product in a time interval of length dt at a Poisson rate of $\lambda + x(t)$, where $\lambda > 0$ is the exogenous rate of learning-by-purchasing and $x(t) \geq 0$ is the advertising intensity that is controlled by the seller. A consumer who does not purchase and consume the product in a time interval of length dt receives a perfectly informative signal at a Poisson rate of $\gamma x(t)$, where $\gamma \in [0, 1]$ measures the degree of substitutability between learning from advertising and consumption experience. For example, television and Internet advertising may correspond to $\gamma = 0$, whereas distributing free samples of the product may correspond to $\gamma = 1$. Advertising that describes the product cannot substitute for consumption experience when it is difficult for buyers to predict how they value the experience attributes of the product without consuming it. On the other hand, if advertising takes the form of distributing free samples of the product, learning from advertising can easily substitute for learning-by-purchasing.

The flow expenditure needed to achieve the rate of advertising intensity $x(t) \in [0, x_h]$ is denoted by $c(x(t))$, where c is a twice-differentiable, strictly increasing, and convex function with $c(0) = c'(0) = 0$ and $c'(x_h) = \infty$, and where $x_h > 0$ is the maximum allowable advertising rate.⁵

At each instant, the monopolist sets a spot price $p(t)$ and an advertising rate $x(t)$. Upon seeing the price and advertising rate, each consumer decides whether to purchase or not.⁶ Then the uninformed consumers observe whether or not they have learned their actual valuation for the product.

Let $\alpha(t) \in [0, 1]$ denote the share of consumers that are informed at time t . The state variable $\alpha(t)$ evolves according to

$$\frac{d\alpha(t)}{dt} = \begin{cases} (\lambda + x(t))(1 - \alpha(t)), & \text{if the uninformed consumers buy in period } t \\ \gamma x(t)(1 - \alpha(t)), & \text{if the uninformed consumers do not buy in period } t \end{cases}, \quad (1)$$

⁵ This advertising cost function has the properties of the “constant-reach, independent readership” technology used in Grossman and Shapiro (1984). Also, as pointed out in Heiman et al. (2001), convex costs of informative advertising via a free sample promotion arise if a firm pursues the cheapest locations or time slots first.

⁶ The assumption that buyers observe the current advertising rate can perhaps be justified by distinguishing between the sent and received advertising messages. For example, a buyer can see whether or not there is an ad in a newspaper, but whether or not the advertisement helps him learn depends on whether he has time to read it.

since at time t there are $1 - \alpha(t)$ currently uninformed consumers and, depending on the purchasing decision of the uninformed buyers, either a fraction $(\lambda + x(t))dt$ or a fraction $\gamma x(t)dt$ of them become informed in a time interval of length dt .⁷

Markovian pricing and advertising strategies for the seller are denoted by $p(\alpha)$ and $x(\alpha)$, respectively. The uninformed buyer has a Markovian purchasing strategy of $d^u(\alpha, p, x)$. The informed buyer with valuation θ also has a Markovian purchasing strategy of $d^\theta(\theta, \alpha, p, x)$. The monopolist maximizes her expected discounted profit over the infinite horizon, and the buyers maximize the (expected) discounted value of their utilities from consumption net of price.

⁷ The probability that a consumption opportunity and an advertising signal simultaneously arrive to an uninformed consumer, $\lambda x(t)(dt)^2$, goes to zero much faster than dt and hence can be ignored.

3. EQUILIBRIUM

We characterize the Markov perfect equilibrium (MPE) of this advertising monopoly game. In such an equilibrium the decisions of the monopolist and the consumers depend only on the current share of informed consumers, $\alpha(t)$, the state variable of the model. As in BV, even though consumers and the seller do not directly observe $\alpha(t)$, they can infer its value from the equilibrium purchasing and advertising strategies using equation (1). To simplify notation, we will omit the indexation with respect to time t until we compare models with and without advertising.

For a given price and given advertising policies, $p(\alpha)$ and $x(\alpha)$ respectively, the value function $V^\theta(\alpha)$ of the informed buyer satisfies the Bellman equation:

$$rV^\theta(\alpha) = \max[\theta - p(\alpha), 0] + \frac{dV^\theta}{d\alpha} \frac{d\alpha}{dt}. \quad (2)$$

Because the purchasing decision of a single consumer has no impact on the seller's profits, the myopic decision rule is optimal for the informed buyer: Buy if $p(\alpha) \leq \theta$. The value function of the uninformed buyers satisfies

$$rV^u(\alpha) = \max[v - p(\alpha) + (\lambda + x(\alpha))(E[V^\theta(\alpha)] - V^u(\alpha)), \gamma x(\alpha)(E[V^\theta(\alpha)] - V^u(\alpha))] + \frac{dV^u}{d\alpha} \frac{d\alpha}{dt}. \quad (3)$$

As in BV, a current purchase generates the (expected) flow of consumption, v , and information, $(\lambda + x(\alpha))(E[V^\theta(\alpha)] - V^u(\alpha))$. Because an uninformed buyer becomes informed even without purchasing the product at rate $\gamma x(\alpha)$, the maximum willingness to pay of the uninformed buyers is given by $v + (\lambda + (1 - \gamma)x(\alpha))(E[V^\theta(\alpha)] - V^u(\alpha))$.

Let $V(\alpha)$ denote the value function of the monopolist's program, and let $\pi(\alpha, p) = (1 - \alpha + \alpha(1 - F(p)))(p - k)$ denote the flow profit (gross of advertising cost) when the uninformed buyers purchase the product. If uninformed buyers participate in the market, the relevant Bellman equation is

$$rV(\alpha) = \max_{p(\alpha), x(\alpha) \in \mathbb{R}_+} \pi(\alpha, p(\alpha)) - c(x(\alpha)) + \frac{dV}{d\alpha} \frac{d\alpha}{dt}, \quad (4)$$

subject to $p(\alpha) \leq v + (\lambda + (1 - \gamma)x(\alpha))(E[V^\theta(\alpha)] - V^u(\alpha))$.

If the monopolist sells only to the informed buyers, the value function satisfies

$$rV(\alpha) = \max_{p(\alpha), x(\alpha) \in \mathbb{R}_+} \alpha(1 - F(p(\alpha)))(p(\alpha) - k) - c(x(\alpha)) + \frac{dV}{d\alpha} \frac{d\alpha}{dt}. \quad (5)$$

Given the purchasing strategies, the seller seeks optimal dynamic pricing and advertising strategies. An MPE of this game is a quadruple (d^u, d^θ, p, x) such that equations (2) through (5) are simultaneously solved for all α and θ .

As in the BV model, the equilibrium pricing policy solves a stopping problem. Initially, all buyers

are uninformed, and the equilibrium spot price must equal their maximum willingness to pay. However, as the learning process continues, the monopolist needs to decide at what point to stop setting the price equal to the willingness to pay of the uninformed. Let

$$\hat{\alpha} = \inf\{\alpha \in [0,1] : p(\beta) \neq v + (\lambda + (1-\gamma)x(\beta))(E[V^\theta(\beta)] - V^u(\beta)) \quad \forall \beta \geq \alpha\}$$

denote the size of the uninformed segment at the stopping point \hat{t} of the pricing policy, where $\alpha(\hat{t}) = \hat{\alpha}$.

It will also be convenient to let

$$\alpha_0 = \inf\{\alpha \in [0,1] : x(\beta) = 0 \quad \forall \beta \geq \alpha\}$$

denote the threshold size of the informed segment such that the seller stops advertising once α reaches that level at t_0 , where $\alpha(t_0) = \alpha_0$.

For a moment let us consider the seller's incentives to advertise in a static model for a fixed willingness to pay of uninformed buyers $w(\alpha)$. Then it is convenient to think of α as parameterizing the family of the aggregate distributions of valuations

$$F_\alpha(\theta) \equiv \begin{cases} \alpha F(\theta), & \text{if } \theta \leq w(\alpha) \\ 1 - \alpha + \alpha F(\theta), & \text{if } \theta > w(\alpha) \end{cases}.$$

This family is ordered by a sequence of rotations around $w(\alpha)$ in the sense of Johnson and Myatt (2006). In other words, as the share of informed buyers increases, the aggregate distribution of willingness to pay becomes more dispersed around $w(\alpha)$. Provided that the rotation point $w(\alpha)$ is decreasing in α , Johnson and Myatt (2006) show that the static optimal monopoly profit is a U-shaped function of α (762, Proposition 1).

This suggests that, all else equal, the seller's incentives to advertise, if at all, should be greater as α increases. However, our dynamic model adds three important considerations that we explore in the analysis to follow. First, the seller controls the evolution of α by means of both advertising policy and price policy. Second, the willingness to pay of uninformed buyers $w(\alpha)$ is endogenous and is determined by the current advertising intensity and the path of future advertising intensities and spot prices. Third, the advertising technology is subject to diminishing returns because it affects only uninformed consumers, whose number falls over time.

Advertising Rate during Early and Mature Stages

Following BV, we refer to a period of time during which the marginal buyer is uninformed (informed) as an early (respectively, mature) stage. By (4), the optimality condition for the advertising rate during an early stage is given by⁸

$$\frac{\partial \pi(\alpha, p(\alpha))}{\partial p(\alpha)} \frac{\partial p(\alpha)}{\partial x(\alpha)} + (1-\alpha) \frac{dV(\alpha)}{d\alpha} - c'(x(\alpha)) \leq 0, \quad (6)$$

where $\frac{\partial p(\alpha)}{\partial x(\alpha)} = (1-\gamma)(E[V^\theta(\alpha)] - V^u(\alpha))$ is the rate at which the willingness to pay of the uninformed buyers (that is, the current spot price) increases due to current advertising. We will refer to $\frac{\partial \pi \partial p}{\partial p \partial x}$ and $(1-\alpha) \frac{dV}{d\alpha}$ as, respectively, the immediate and long-run effects of advertising. The immediate effect of advertising is positive when the constraint in (4) is binding and advertising complements experience

⁸ We assume that either x_h is sufficiently small or θ_h is sufficiently large so that the spot price is less than θ_h and the flow profit function $\pi(\alpha, p(\alpha))$ is differentiable.

($\gamma < 1$), but otherwise it is null. However, the long-run effect can be positive or negative depending on whether future profits are an increasing or decreasing function of the size of the informed segment.

If the uninformed buyers continue to purchase the product during a mature stage, advertising has no marginal effect on the current profits because the constraint in (4) is slack, and the optimality condition for the advertising rate becomes

$$(1 - \alpha) \frac{dV(\alpha)}{d\alpha} - c'(x(\alpha)) \leq 0. \quad (7)$$

Finally, if the uninformed buyers are not purchasing the product during a mature stage, by (1) and (5), the optimality condition for the advertising rate is given by

$$\gamma(1 - \alpha) \frac{dV(\alpha)}{d\alpha} - c'(x(\alpha)) \leq 0. \quad (8)$$

Given that advertising is less likely to provide information to nonpurchasing buyers, the effect of advertising on the rate of change of the monopolist's value function is smaller in (8) than in (7). Before we study in more detail how the incentives to advertise evolve over time, let us discuss the determinants of the price policy.

Introductory Prices

During the early stages, the path of equilibrium spot prices is governed by the following differential equation:

$$\frac{dp}{d\alpha} \frac{d\alpha}{dt} = (r + \gamma x(\alpha) + \frac{1-\gamma}{\lambda + (1-\gamma)x(\alpha)} \frac{dx}{d\alpha} \frac{d\alpha}{dt})(p(\alpha) - v) - (\lambda + (1-\gamma)x(\alpha))E \max[\theta - p(\alpha), 0]. \quad (9)$$

For a given change in the advertising rate, the change in the spot price is such that the uninformed buyer is indifferent between purchasing today and purchasing tomorrow. On the one hand, a purchase today generates information, which allows the buyer to make an informed decision, at an incremental rate of $\lambda + (1-\gamma)x(\alpha)$ relative to the rate at which an informative signal arrives to a nonpurchasing buyer. On the other hand, in addition to the changes in the price and the advertising rate, buying tomorrow reduces the present cost of information, $p(\alpha) - v$, and allows the buyer to learn from advertising without purchasing at rate $\gamma x(\alpha)$. Next we discuss how the stopping point $\hat{\alpha}$ is determined.

Switching Point of Price Policy and Prices during the Mature Stage

Following BV's notation, we let

$$\hat{p} = \arg \max_{p \in \mathbb{R}_+} (1 - F(p))(p - k)$$

denote the price that maximizes the flow profits from the informed buyers, and we let

$$\hat{w} = v + \frac{\lambda}{r} E \max[\theta - \hat{p}, 0]$$

denote the willingness to pay of the uninformed consumers when there is no advertising in the future and the price remains constant at \hat{p} . As in the model without advertising, the uninformed buyers continue purchasing during the mature stages if $\hat{w} \geq \hat{p}$ (a mass market) but stop purchasing if $\hat{w} < \hat{p}$ (a niche market).

In a mass market, once α reaches $\hat{\alpha}$, as in the model without advertising, the uninformed buyers

do not stop purchasing the product. As in the BV model, for all $\alpha \geq \hat{\alpha}$ the spot price maximizes the flow profits with a slack constraint in (4), that is, $p(\alpha) = \arg \max_p \pi(\alpha, p)$, which implies that $p(\alpha)$ falls over time during the mature stage. During the mature stage, the information gain attached to a current purchase, $(\lambda + (1 - \gamma)x(\alpha))(EV^\theta(\alpha) - V^u(\alpha))$, decreases at a slower rate than the spot price, and the optimal switching point of the price policy is the smallest $\hat{\alpha}$ such that

$$\frac{\partial \pi(\hat{\alpha}, p(\hat{\alpha}))}{\partial p} = 0, \quad (10)$$

where $p(\hat{\alpha}) = v + (\lambda + (1 - \gamma)x(\hat{\alpha}))(E[V^\theta(\hat{\alpha})] - V^u(\hat{\alpha}))$.

In contrast, in a niche market, once α reaches $\hat{\alpha}$ the uninformed consumers stop purchasing the product, and the price is permanently set at \hat{p} . At $\hat{\alpha}$ the seller must be indifferent between attracting the uninformed consumers for the last time and switching to a higher price.⁹ The payoff from attracting the uninformed consumers for the last time is the flow profit minus the advertising cost plus the gain in the future profits from a larger segment of the informed consumers due to learning-by-purchasing that is facilitated by advertising:

$$\lim_{\alpha \uparrow \hat{\alpha}} \pi(\alpha, p(\alpha)) - c(x(\alpha)) + (x(\alpha) + \lambda)(1 - \alpha) \frac{dV(\alpha)}{d\alpha}, \quad (11)$$

where $\lim_{\alpha \uparrow \hat{\alpha}} p(\alpha) = \lim_{\alpha \uparrow \hat{\alpha}} v + (\lambda + (1 - \gamma)x(\alpha))(EV^\theta(\alpha) - V^u(\alpha))$. The payoff from offering \hat{p} is the flow profit minus the advertising cost plus the gain in the future profits from a larger informed segment due to learning from advertising by the nonpurchasing uninformed consumers:

$$\lim_{\alpha \downarrow \hat{\alpha}} \alpha(1 - F(\hat{p}))(\hat{p} - k) - c(x(\alpha)) + \lambda x(\alpha)(1 - \alpha) \frac{dV(\alpha)}{d\alpha}. \quad (12)$$

And so, at $\hat{\alpha} \in [0, 1]$ the payoff in (12) must be greater than or equal to the payoff in (11).

⁹ Because the uninformed buyers stop purchasing after $\hat{\alpha}$, both the price and advertising paths are discontinuous at $\hat{\alpha}$. The monopolist may skip the period of introductory prices; that is, we may have $\hat{\alpha} = 0$ if the average consumer valuation net of production cost is sufficiently low.

4. STOPPING OF ADVERTISING IN MASS AND NICHE MARKETS

We first consider when advertising stops in a mass market.

PROPOSITION 1 (Equilibrium stopping of advertising in the mass market). If $\hat{w} \geq \hat{p}$, then

1. $\alpha_0 < \hat{\alpha} \leq 1$;
2. α_0 is decreasing in γ if $\alpha_0 > 0$;
3. $\alpha_0 = 0$ if γ is sufficiently close to 1; and
4. $\alpha_0 > 0$ if γ is sufficiently close to 0 and $v - k \leq (1 - F(\hat{p}))(\hat{p} - k)$.

In a mass market, advertising is offered in an early phase if the positive immediate effect dominates the negative long-run effect. We find that the marginal gain in the current profits due to advertising temporarily offsets the loss due to the acceleration in the decline of the future profits whenever advertising speeds up idiosyncratic learning, mostly in combination with concurrent consumption of the product, and whenever the expected flow of net utility from consumption is small. However, advertising always stops before α reaches $\hat{\alpha}$, because when the marginal buyer is informed, advertising would, in fact, decrease future profits because the uninformed segment generates relatively more sales. All else equal, advertising stops sooner if the degree of substitutability between learning from purchasing and advertising is greater because then the positive immediate effect in equation (6) is weaker. In fact, if advertising is a sufficient substitute for learning-by-purchasing, the equilibrium advertising policy is not to advertise at all.

In a niche market, an increase in the degree of substitutability between learning from purchasing and advertising has the opposite effect on the stopping point of the advertising policy. Now the seller advertises during the mature stage as long as advertising provides information to nonpurchasing consumers. If we compare a mass and a niche market, it is worth pointing out that pure complementary advertising is discontinued for distinct reasons. In a mass market, even though advertising raises the spot price at which the seller can attract the uninformed buyers, consumer learning reduces future profits. In a niche market, however, learning increases profits in the long run, and advertising stops only when the uninformed consumers are priced out of the market.

PROPOSITION 2 (Equilibrium stopping of advertising in the niche market). If $\hat{w} < \hat{p}$, then

1. $\alpha_0 = \hat{\alpha} < 1$, if $\gamma = 0$; and
2. $\alpha_0 = 1$, if $\gamma > 0$.

Note that, for a fixed price above the production cost, the flow profit is decreasing in α when uninformed buyers purchase the product, but it is increasing in α when they do not. We should remark that, in a niche market, the monopolist's value function begins to increase before α reaches $\hat{\alpha}$. To see why, let us rewrite the monopolist's value function in equation (4) in a more explicit form as

$$V(\alpha(t)) = \int_t^{\hat{\alpha}} e^{-r(s-t)} [\pi(\alpha(s), p(\alpha(s)) - c(x(\alpha(s))))] ds \\ + \int_{\hat{\alpha}}^{\infty} e^{-r(s-t)} [\alpha(s)(1 - F(\hat{p}))(\hat{p} - k) - c(x(\alpha(s)))] ds$$

The value function, being the sum of the discounted future profits, inherits the monotonicity properties of the flow profit function but in a “smoothed out” fashion. Using the terminology of Johnson and Myatt (2006), we can say that $V(\alpha)$ decreases because consumer learning erodes the benefits of “mass-market posturing,” and it increases as the monopolist prepares to take a more profitable “niche

posture,” that is, to start pricing the uninformed buyers out of the market. In a mass market, Johnson and Myatt’s (2006) mass-market posturing is the more profitable strategy throughout the marketing cycle, whereas in a niche market, Johnson and Myatt’s (2006) niche posture may be initially the less profitable strategy, but it becomes the more profitable one in the long run.

5. ADVERTISING INTENSITY

Next we will explore in more detail the evolution of the equilibrium advertising intensity. In an early phase the equilibrium advertising rate satisfies the following differential equation:

$$c''(x(\alpha)) \frac{dx(\alpha)}{d\alpha} \frac{d\alpha}{dt} = \frac{dS(\alpha)}{d\alpha} \frac{d\alpha}{dt} + r(c'(x(\alpha)) - S(\alpha)) - (1 - \alpha)L(\alpha), \quad (13)$$

where $S(\alpha) = (1 - \gamma)(EV^\theta(\alpha) - V^u(\alpha)) \frac{\partial \pi(\alpha, p(\alpha))}{\partial p(\alpha)}$ is the marginal effect of advertising on the current profit (that is, the immediate effect) and $L(\alpha) = \frac{\partial \pi(\alpha, p(\alpha))}{\partial \alpha} + ((1 - \gamma)x(\alpha) + \lambda) \times \frac{\partial \pi(\alpha, p(\alpha))}{\partial p(\alpha)} \frac{\partial (EV^\theta(\alpha) - V^u(\alpha))}{\partial \alpha}$ is the change in the flow profits keeping the current rate of advertising constant, so that $L(\alpha) + S(\alpha) \frac{dx(\alpha)}{d\alpha} = \frac{d\pi(\alpha, p(\alpha))}{d\alpha}$.

It is easier to see the intuition behind (13) if we integrate it over a small period of time Δ as follows:

$$\int_t^{t+\Delta} e^{-r(s-t)} (1 - \alpha(s))L(\alpha(s))ds + S(\alpha(t)) - c'(x(\alpha(t))) = e^{-r\Delta} [S(\alpha(t + \Delta)) - c'(x(\alpha(t + \Delta)))]. \quad (14)$$

According to (14), along the equilibrium path, the seller cannot increase profits by shifting the time of offering a marginal bit of advertising. The marginal discounted profit from advertising at time $t + \Delta$ (the right-hand side) must equal the marginal discounted profit achieved by an earlier offering of that advertising at t (the left-hand side). The marginal bit of advertising that is shifted from $t + \Delta$ to t directly generates the additional profit $S(\alpha(t))$ net of cost $c'(x(\alpha(t)))$ as well as indirectly affecting future profits at rate $(1 - \alpha(t)) L(\alpha(t))$ during the period from t to $t + \Delta$. Finally, at $t + \Delta$, the seller forgoes the discounted incremental profit $S(\alpha(t + \Delta))$ and saves the incremental cost $c'(x(\alpha(t + \Delta)))$. To say more about the behavior of the advertising policy, we consider two cases with low and high values of γ .

6. ADVERTISING THAT COMPLEMENTS LEARNING-BY-PURCHASING

Now we assume that advertising provides no useful information about the product to nonpurchasing consumers.¹⁰ We begin with the case of a mass market. The following result identifies conditions under which advertising starts when the product is first introduced and the advertising intensity decreases over time.

PROPOSITION 3. If $\hat{w} \geq \hat{p}$, $\gamma = 0$ and $v - k \leq (1 - F(\hat{p}))(\hat{p} - k)$, then there exists $r^* > 0$ such that for any $r \leq r^*$ the advertising rate $x(\alpha)$ is strictly positive and decreasing for all $0 \leq \alpha < \alpha_0$.

As the uninformed segment shrinks, the magnitudes of both the positive immediate and negative long-run effects of advertising decline. Yet when the discount factor is small, the long-run effect weakens more slowly than the immediate effect, and the returns on advertising diminish over time. Figure 1 graphically depicts the declining paths of advertising and prices in a mass market.¹¹ The price, shown as the solid line in Figure 1b, decreases over time because both the rate of learning complemented by advertising, $\lambda + x(\alpha)$, and the expected discounted future loss of utilities from paying the price that exceeds the actual valuation, $EV^\theta(\alpha) - V^u(\alpha)$, decline over time.

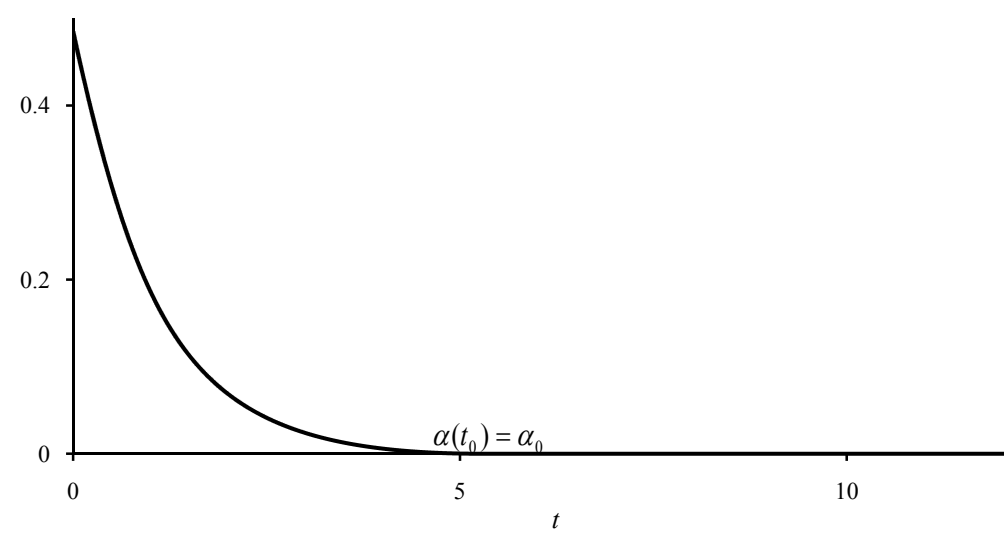
Conversely, if the negative long-run effect weakens faster than the positive immediate effect, the marginal returns on advertising increase over time. Then during the early stages the spot price may increase as well. Uninformed buyers need an additional incentive to purchase today rather than tomorrow when a delayed purchase generates more learning due to more intensive advertising as well as more valuable information due to temporarily higher future prices.¹²

¹⁰ Even though this case may seem special, nonpurchasing buyers may, in fact, learn very little about their preferences from ad information without having direct experience with the product.

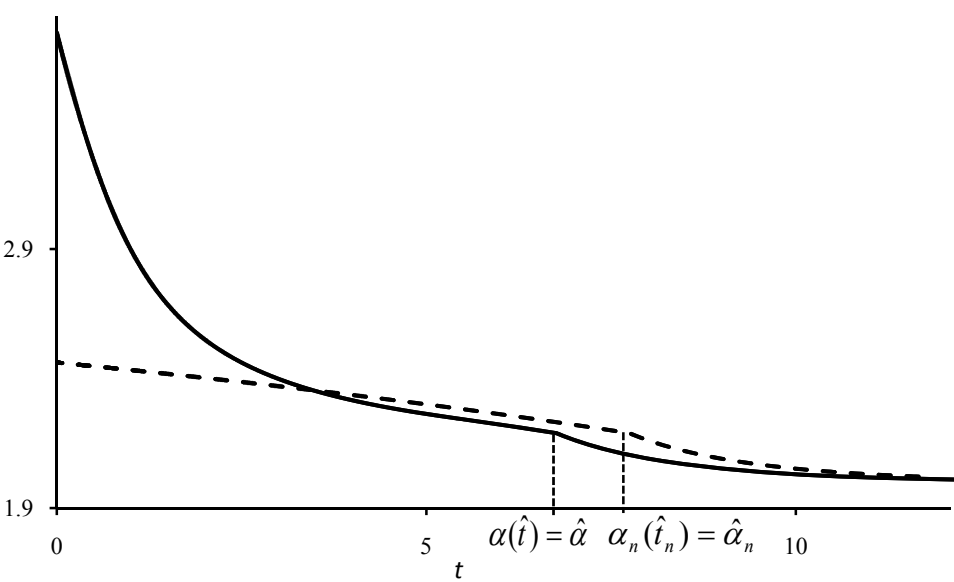
¹¹ Figure 1 depicts the advertising and price paths for the following values of parameters: $F(\theta) = e^{-\theta}$ for $\theta \geq 0$, $r = 0.01$, $\lambda = 0.5$, $k = 1$, and $c(x) = 0.5x^2$.

¹² For example, the advertising and price paths are increasing during the early stages in a mass market when θ takes three values: $\theta_0 = 0$, $\theta_1 = 1$, and $\theta_2 = 2$, with probabilities $f_0 = 0.32$, $f_1 = 0.5$, and $f_2 = 0.18$, respectively; $k = 0.5$, $c(x) = 10x^2$, $r = 0.01$, and $\lambda = 0.05$.

Figure 1. Advertising and price paths for the mass market



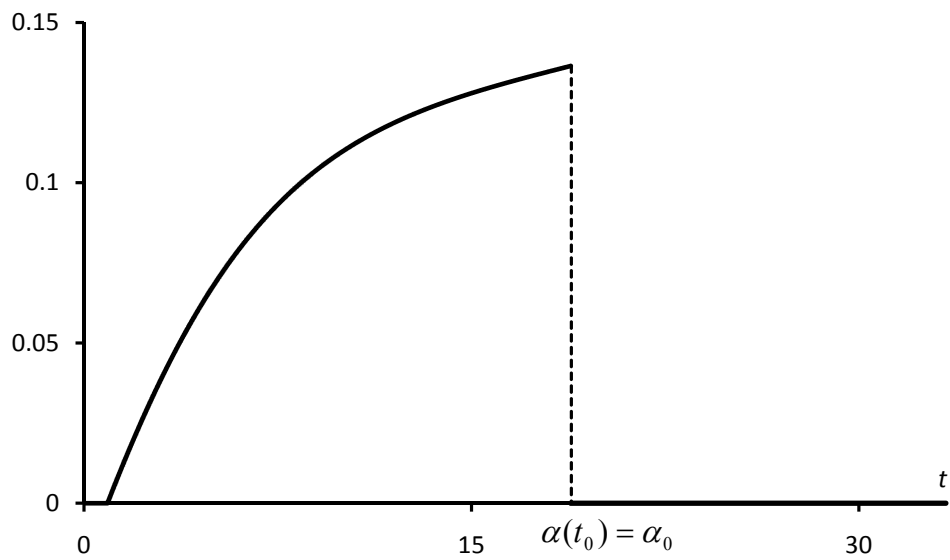
a. Advertising path



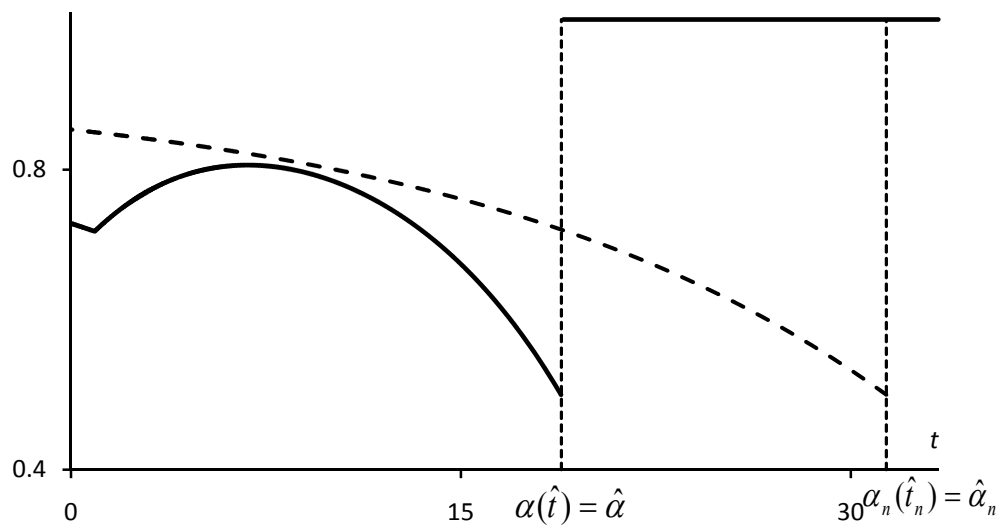
b. Price paths

Source: Simulations for specific parameter values.

Figure 2. Advertising and price paths for the niche market for $k = 0$



a. Advertising path



b. Price paths

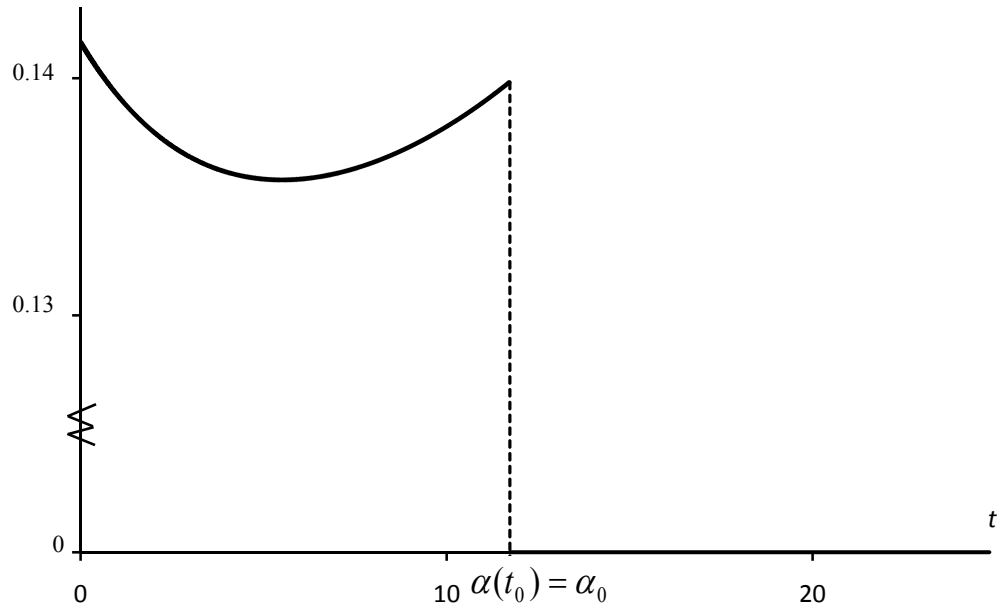
Source: Simulations for specific parameter values.

Next we consider the case of a niche market (with $\gamma = 0$). We explore the different ways in which the sum of the immediate and long-run effects of advertising evolves over time using a particular example of a niche market with a binary distribution of tastes and quadratic advertising cost function for different values of the production cost.¹³ When the production cost is sufficiently low ($k < 0.4946$), the advertising rate is rising over time in an early market, but advertising may be delayed as, initially, the negative long-run effect of advertising may dominate the positive immediate effect. For example, in Figure 2a, advertising commences at $t_0 = 0.909$ when the positive immediate effect just offsets the negative long-run effect, where $\alpha(t_0) = 0.0868$. The increasing long-run effect subsequently compensates for the weakening immediate effect, so that the advertising intensity is growing during the remainder of the early market until $\hat{t} = 18.848$. At \hat{t} , the seller stops subsidizing the learning process by offering low prices and complementary advertising, and raises the price in order to extract surplus from buyers with a high willingness to pay. As we discussed earlier, even though the flow profits decrease in the early stages, the monopolist's value function begins to increase before the end of the period of introductory prices as the time approaches when the seller reaps the long-run benefits of consumer learning. Specifically, in the example in Figure 2, $dV/d\alpha < (>)0$ as $t < (>)17.345$.

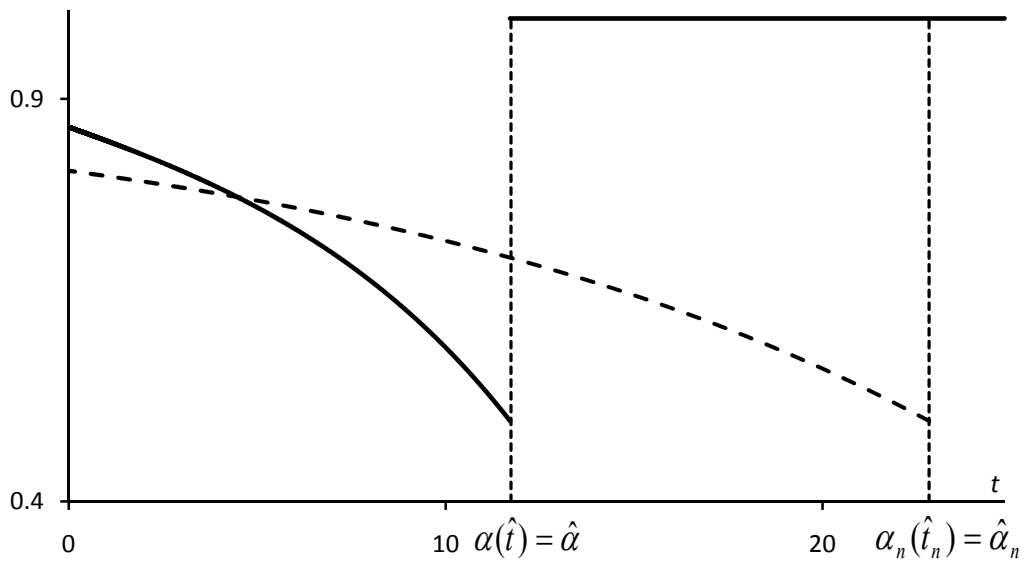
A higher production cost not only reduces the flow profit but also increases the wedge between the willingness to pay of the uninformed buyers and the static optimal price. This tends to boost the immediate effect of advertising relative to its long-run effect at $t = 0$. As a result, for the production cost in an intermediate range ($0.4946 \leq k < 0.75$), the advertising path is U-shaped during the period of introductory prices. For example, in Figure 3a, the marginal returns on advertising temporarily decline and bottom out at $t = 5.503$ when the immediate effect has already weakened and the long-run effect of advertising is yet to strengthen. Finally, if the production cost is sufficiently high ($k \geq 0.75$), the advertising rate is falling over time because the diminishing immediate effect of advertising dominates the long-run effect throughout the early phase.

¹³ Here we assume that θ has a binary distribution with $\Pr(\theta=0) = \Pr(\theta=1) = 0.5$, $c(x) = 5x^2$, $r = 0.01$, and $\lambda = 0.5$. To simplify the presentation we vary the production cost k rather than the distribution of tastes $F(\theta)$. The effect of the production cost (or the shift in the mean of the distribution of tastes) on the pattern of the advertising policy is similar in the case of uniform, normal, and exponential distributions. One noteworthy difference, however, is that the price path tends to follow the advertising path more closely when the information rent for the informed buyers does not vanish over time.

Figure 3. Advertising and price paths for the niche market for $k = 0.55$



a. Advertising path



b. Price paths

Source: Simulations for specific parameter values.

7. ADVERTISING THAT SUBSTITUTES FOR LEARNING-BY-PURCHASING

Now we investigate what happens when learning from advertising and from purchasing are good substitutes for each other, as in the case of free product sampling. We continue to assume that ad exposures, by themselves, provide buyers with zero flow utility of consumption.¹⁴ To see how the incentives to advertise change when advertising provides information about individual tastes to both purchasing and nonpurchasing buyers, we consider a polar case with $\gamma = 1$. In this case the immediate effect of advertising is null because the willingness to pay of the uninformed buyers is independent of the current advertising rate. As a result, the marginal returns on advertising are determined solely by its long-run effect during both early and mature stages.

By Propositions 1 and 2, we need only to consider the case of a niche market. The following characterization of the equilibrium paths of advertising rate and spot prices relies on the assumptions that the advertising cost function is sufficiently convex, in the sense that the maximum allowable advertising rate x_h is sufficiently small, and that $v \geq k$, which assures that the introductory prices exceed the production cost.¹⁵

PROPOSITION 4. If $\hat{w} < \hat{p}$, $\gamma = 1$, and $v - k \geq 0$, then there exists x_h^* such that for all $x_h \leq x_h^*$,

1. the advertising rate $x(\alpha)$ is increasing for all $0 \leq \alpha < \hat{\alpha}$ and is strictly decreasing for all $\hat{\alpha} < \alpha < 1$, and if $v - k \geq (1 - F(\hat{p}))(\hat{p} - k)$, then there exists $\alpha^* \in (0, \hat{\alpha})$ such that $x(\alpha) = 0$ for all $0 \leq \alpha \leq \alpha^*$ and $x(\alpha)$ is strictly increasing for all $\alpha^* \leq \alpha < \hat{\alpha}$; and
2. the price $p(\alpha)$ is strictly decreasing for all $\alpha < \hat{\alpha}$ and then jumps up and stays at $p(\alpha) = \hat{p}$ for all $\alpha > \hat{\alpha}$.

For $\gamma = 1$ the differential equation (13) that governs the evolution of the advertising intensity during the early stages, upon rearrangement, becomes

$$(1 - \alpha) \frac{d\pi(\alpha, p(\alpha))}{d\alpha} = rc'(x(\alpha)) - c''(x(\alpha)) \frac{dx}{d\alpha} \frac{d\alpha}{dt}.$$

In this form it admits a straightforward interpretation. The left-hand side represents the net (negative) payoff from one more bit of advertising today rather than tomorrow. It depends on the size of the uninformed segment, $1 - \alpha$, and the total change in the flow profits due to the marginal increase in the number of informed buyers, $d\pi/d\alpha$. The right-hand side represents the net (negative) payoff from one more bit of advertising tomorrow rather than today. It consists of the savings from delaying the additional advertising expenditures, rc' , and the change in the marginal advertising cost tomorrow, dc'/dt . During the early stages the net payoff from the marginal bit of advertising today rather than tomorrow is negative because, given the price policy, the uninformed buyers are more lucrative customers than informed buyers. Therefore, because in equilibrium the seller is indifferent between shifting the marginal bit of advertising between today and tomorrow, delaying advertising must generate net “cost dissaving” due to higher marginal costs of advertising in the future. This happens only when the advertising rate is rising over time.

Seeing that the current advertising rate has no impact on the current willingness to pay (because consumers learn from advertising at the same rate whether they purchase or not), the evolution of the spot

¹⁴ An example suggested by a referee is that it can be inconvenient to collect many small samples of soap powder to substitute for one large pack. So if the relevant good is one pack, free samples have little consumption value.

¹⁵ If advertising takes the form of product sampling, it may be reasonable to assume that $c(x) = (x/\lambda)k$ and $x_h \leq \lambda$, since samples are typically smaller than the actual product.

price can now be easily understood as being driven by two considerations. On the one hand, as the period of low introductory prices draws to an end, the option value generated by the information may decrease because of the upcoming change in the price policy that will make the product more expensive. On the other hand, as the advertising intensity grows, the willingness to pay of the uninformed buyers may increase because they are more willing to delay purchasing when there is a greater likelihood of receiving a free informative advertising message in the future. When the advertising is not too intense, the smaller-future-information-rent effect dominates the waiting-to-learn-for-free effect, and the price falls over time.¹⁶

During the mature stage, after α has reached $\hat{\alpha}$, the qualitative properties of equilibrium do not change if we allow for any $\gamma > 0$ (including $\gamma = 1$). Once the uninformed consumers stop purchasing, advertising continues but at a decreasing rate, provided that the nonpurchasing buyers can learn from advertising. Along the equilibrium advertising path, the marginal advertising cost must equal the present value of the profits that the monopolist forgoes in the future because some consumers remain uninformed:

$$c'(x(\alpha(t))) = \gamma \int_t^{\infty} e^{-r(s-t)} (1 - \alpha(s))(1 - F(\hat{p}))(\hat{p} - k) ds \text{ for all } \alpha(t) > \hat{\alpha}.$$

As time goes by, there are fewer uninformed consumers left in the population, and the marginal returns on advertising (the right-hand side) diminish. So the equilibrium advertising intensity reaches its highest point during the early stages for any $\gamma \in [0,1]$.

¹⁶ If x_h is sufficiently large, the spot price may rise and the advertising rate may fall before $\hat{\alpha}$ is reached.

8. THE EFFECT OF ADVERTISING ON LEARNING AND PRICES

Now we examine how advertising affects purchasing behavior, aggregate learning, and prices by comparing them in equilibrium in models with and without advertising. We will use the subscript n to denote the corresponding endogenous variables in equilibrium in the BV model without advertising. Because, generally, the size of the informed segment evolves differently in the two models, we will use the time coordinate t rather than α as an independent variable.

PROPOSITION 5 (Advertising, switching points of the price policy, and learning).

1. If $\hat{w} \geq \hat{p}$, then $\hat{\alpha} = \hat{\alpha}_n$.
2. If $\hat{w} < \hat{p}$, then $\hat{\alpha} > \hat{\alpha}_n$ for $\gamma=0$, and $\hat{\alpha} < \hat{\alpha}_n$ for $\gamma=1$.
3. There exists $t^* \in [0, \infty)$ such that $\alpha(t) > \alpha_n(t)$ for all $t > t^*$.

In a mass market, the switching point of the price policy $\hat{\alpha}$ is determined independently of the advertising policy because advertising stops before the switching point is reached and, consequently, $\hat{\alpha}$ coincides with $\hat{\alpha}_n$. Also, because the uninformed buyers are always purchasing, advertising necessarily accelerates learning, that is, $\alpha(t) \geq \alpha_n(t)$ for all $t \geq 0$. In a niche market, the comparison between $\hat{\alpha}$ and $\hat{\alpha}_n$ depends on the value of γ . If advertising provides no (or little) information to nonpurchasing buyers, the threshold size of the informed segment at which the uninformed buyers stop purchasing is greater vis-à-vis that in equilibrium without advertising. Intuitively, the seller is more willing to “subsidize” learning-by-purchasing when experiential learning is enhanced by advertising. However, because advertising accelerates the learning process, the period of time during which the introductory prices are offered can be either shorter or longer, that is, $\hat{t} < (\geq) \hat{t}_n$, depending on whether the advertising intensity is high or low.

If, on the other hand, advertising generates the same (or a similar) amount of learning for nonpurchasing and purchasing buyers, the threshold size of the informed segment at which the uninformed buyers stop purchasing is smaller, and the period of time during which introductory prices are offered is shorter, that is, $\hat{t} < \hat{t}_n$. As a result, advertising crowds out learning-by-purchasing during the time interval between \hat{t} and \hat{t}_n , and the learning process may temporarily slow down if learning-by-purchasing generates information at a higher rate than learning-from-advertising does. Nonetheless, as advertising continues during the mature phase, it raises the size of the informed segment in the long run.

Next we discuss the effect of advertising on prices. Although advertising in this model directly affects prices only during the early stages, its intertemporal effects are quite nuanced. Current advertising (weakly) increases the effectiveness of learning-by-purchasing, which has a positive effect on the willingness to pay of the uninformed buyers. However, future advertising may reduce the gain from becoming informed today because it reduces the likelihood of remaining uninformed in the future. Future advertising may also increase the gain from becoming informed today because with higher future prices it is more likely that an uninformed consumer will purchase the product at a price that exceeds his actual valuation. However, if the uninformed consumers stop purchasing, higher future prices reduce the present value of information. Additionally, advertising influences the price policy through its effect on the rate of aggregate learning and the length of the period during which the introductory prices are offered.

Figure 1b depicts the paths of prices in equilibrium, with and without advertising, in a mass market (the price in equilibrium without advertising is the dotted line). With the advertising intensity diminishing over time, prices that are accompanied by advertising are higher when the product is first introduced but lower later on due to a faster-growing informed segment. However, advertising that intensifies over time may lead to lower prices when the product is launched vis-à-vis those in the

equilibrium without advertising.¹⁷ A general property is that, although informative advertising temporarily raises prices at some point during the early phase in a mass market, it reduces them in the long run; that is, there exists $t_1 \in (0, t_0)$ such that $p(t) > p_n(t)$ for some $t < t_1$ and $p(t) < p_n(t)$ for all $t \geq t_1$.

In a niche market, the influence of advertising on prices is shorter-lived as the seller sets the price at \hat{p} before all consumers are informed. As Figures 2b and 3b illustrate, advertising that generates little learning for nonpurchasing buyers can either raise or lower prices during the early stages. It can be shown that if advertising generates a similar amount of learning for purchasing and nonpurchasing buyers, then $p(t) < p_n(t)$ for all $t < \hat{t}$ and $p(t) = \hat{p} \geq p_n(t)$ for all $t > \hat{t}$. Advertising lowers prices during the shorter introductory period because it provides information regardless of purchasing decisions, which reduces the value of information that is gained by purchasing.

¹⁷ For instance, this is the case for a mass market in the example in footnote 12.

9. ADVERTISING AND WELFARE

We are now in a position to study the effects of advertising on welfare. Let $CS(t) = (1 - \alpha(t))V^u(\alpha(t)) + \alpha(t)EV^\theta(\alpha(t))$ and $W(t) = CS(t) + V(\alpha(t))$ denote the average consumer's and social value functions, respectively. Again, the subscript n denotes the corresponding variables and value functions in equilibrium without advertising in the BV model.

PROPOSITION 6 (Advertising and consumer welfare).

1. $CS(0) \geq CS_n(0)$, and the inequality is strict if either (i) $\gamma = 0$ and $\hat{w} \geq \hat{p}$, or (ii) $\gamma > 0$.
2. There exists $\tilde{t} \in [0, \infty)$ such that $CS(t) \geq CS_n(t)$ for all $t \geq \tilde{t}$.

Because uninformed consumers share the same prior beliefs in this model, the monopolist is able to extract their entire surplus during the early stages when the marginal buyer is uninformed. Thus, the effect of advertising on consumer welfare in the beginning, when all consumers are uninformed, is determined by its long-term effects during the mature stage. Consequently, in the beginning, advertising makes all consumers better off because the long-run prices are (weakly) lower in equilibrium with advertising, and consumers learn faster. In particular, in a mass market the uninformed buyers gain from advertising as they become inframarginal buyers who retain a positive information rent sooner. If advertising provides some information to nonpurchasing buyers, the uninformed buyers are also strictly better off simply because they retain a positive expected information rent during the early and mature stages even without buying the product. And so, while advertising may temporarily raise prices and make the informed buyers worse off, it improves consumer welfare in the long run.

What about the effect of advertising on overall welfare? Although the present value of consumer surpluses is greater, the present value of social welfare can decrease as a result of advertising. This happens because the monopolist cannot commit to future paths of advertising and prices. As a result, future advertising may make information less valuable for the currently uninformed buyers and thus temporarily reduce their surplus from trade. We can rule out such adverse effects of advertising on the seller's profits and social welfare in a niche market when the discount rate is sufficiently small. Also, in a mass market, advertising benefits the seller and increases the present value of social welfare at $t = 0$ provided that the seller stops advertising sufficiently soon.

PROPOSITION 7. If $\hat{w} \geq \hat{p}$ and $v - k \leq (1 - F(\hat{p}))(\hat{p} - k)$, then there exist γ_1 and γ_2 such that $V(0) \geq V_n(0)$ and $W(0) \geq W_n(0)$ for all $\gamma \in [\gamma_1, \gamma_2]$, where $0 \leq \gamma_1 < \gamma_2 < 1$.

In a niche market, as we discussed above, in the long run there are more informed and participating consumers, and welfare improves as a result of advertising. In a mass market, the acceleration of learning has two potentially competing effects on welfare. On the one hand, prices fall more rapidly and generate more socially beneficial trades over time. On the other hand, there are fewer uninformed consumers, and there may be less socially beneficial trading. A larger size of the informed segment may lead to fewer socially beneficial trades because the participation rate of the informed buyers is lower than that of the uninformed buyers. Yet if the forgone social flow surplus from trade with the informed consumers, $\int_{\theta_1}^{\hat{p}} (\theta - k) dF(\theta)$, is negative in the long run, learning eventually generates more socially efficient trades than it destroys, and advertising improves social welfare.¹⁸

PROPOSITION 8 (Advertising and welfare in the long run). If (i) $\hat{w} < \hat{p}$, or if (ii) $\hat{w} \geq \hat{p}$ and $k \geq E[\theta \mid \theta \leq \hat{p}]$, then there exist a $\bar{t} \in [0, \infty)$ such that $W(t) \geq W_n(t)$ for all $t \geq \bar{t}$.

¹⁸ Note that $\int_{\theta_1}^{\hat{p}} (\theta - k) dF(\theta) \geq 0$ is implied by $v - k \leq (1 - F(\hat{p}))(\hat{p} - k)$.

10. CONCLUSION

In this paper, we have analyzed the advertising and pricing policies of a monopolist in a model of advertising that helps consumers learn their valuation for an experience good. We have shown that in a mass market with sufficiently low discount rate and net average consumer valuation, the equilibrium advertising intensity falls over time. However, the monopolist does not necessarily advertise more in the beginning when most consumers are in the process of learning their preferences for the good. In particular, in a niche market, the path of advertising intensity can be U-shaped during the early stages, or bell-shaped depending on whether advertising complements or substitutes for learning-by-purchasing.

We have also compared the learning outcomes, prices, and welfare in equilibrium with advertising vis-à-vis equilibrium without advertising. Although advertising that provides little information to nonpurchasing buyers always accelerates aggregate learning, advertising that provides information to all buyers may lead to less learning-by-purchasing and temporarily slow down the learning process. The effect of advertising on prices is ambiguous. A general property is that advertising increases prices at least for some time before the equilibrium price converges to the long-run price. Nonetheless, advertising raises the present value of consumer welfare because uninformed buyers are more likely to become informed, while any effects of advertising on prices eventually vanish. Yet the present value of social surpluses may be reduced by advertising due to excessive advertising in the future that decreases the value of information today.

Our modeling strategy can also be used to investigate the evolution of an optimal marketing mix between advertising that substitutes and that complements learning-by-purchasing. This can be done by letting the advertising cost $c(x, \gamma)$ be a function of the advertising rate x and the degree of substitutability between learning from advertising and from purchasing γ , and allowing the seller to choose γ and send advertising messages with different levels of γ . Our analysis suggests that advertising that complements learning-by-purchasing will be used in mass markets. However, in the case of a niche market, there will be a shift from advertising that complements experience to advertising that substitutes for learning-by-purchasing as the market matures. An empirical investigation of how the intensity and content of informative advertising for newly launched nondurable experience goods change over time is desirable (Anderson et al. 2010).

An important limitation of the model is that signals obtained from both consumption experience and advertising perfectly reveal true valuations for the product. Clearly, a more realistic assumption is that not only the speed of acquiring but also the precision of information about individual valuations varies depending on whether a buyer learns by consuming the product or by exposure to advertisements. The latter typically convey information about some but not all of the good's experience characteristics via product specifications, descriptions of content or likely users, illustrative analogies, and the like. Allowing consumers to acquire partial information about their true tastes, that is, learn slowly not only at the aggregate but also at an individual level, is an interesting topic for future research.

APPENDIX: PROOFS

It will be convenient to let

$$\hat{\pi} = (1 - F(\hat{p}))(\hat{p} - k) ,$$

and

$$p^m(\alpha) = \arg \max_p \pi(\alpha, p) .$$

Lemma 1 summarizes the equilibrium properties of the BV model (see Bergemann and Valimaki 2006, 724–727, Propositions 1–4) that we will need in our proofs.

Lemma 1 (Bergemann and Valimaki [2006] model). In equilibrium without advertising,

$$1. \quad \frac{dp_n}{d\alpha_n} \lambda(1 - \alpha_n) = r(p_n(\alpha_n) - v) - \lambda E \max[\theta - p_n(\alpha_n), 0] < 0 \text{ for all } \alpha_n ;$$

2. in a niche market, $\hat{\alpha}_n < 1$, and it satisfies the following equation:

$$g_n(\hat{\alpha}_n) \equiv \pi(\hat{\alpha}_n, \hat{w}) + \frac{\hat{\pi}}{r} \lambda(1 - \hat{\alpha}_n) - \hat{\alpha}_n \hat{\pi} = 0 ; \text{ and}$$

3. in a mass market, $\hat{\alpha}_n \leq 1$ is increasing in r , and

$$\frac{d\pi(\alpha_n, p_n(\alpha_n))}{d\alpha_n} < 0 , \frac{dV_n(\alpha_n)}{d\alpha_n} < 0 \text{ for all } \alpha_n < 1 , \text{ and}$$

$$p_n(\hat{\alpha}_n) = v + \lambda^2 \int_{\hat{i}_n}^{\infty} \int_{\hat{i}_n}^T e^{-r(s - \hat{i}_n)} E \max[p^m(\alpha(t)) - \theta, 0] dt e^{-\lambda(T - \hat{i}_n)} dT .$$

Lemma 2. In equilibrium, $p(\alpha) = p_n(\alpha)$ for all $\alpha > \alpha_0$.

Proof of Lemma 2

Perfection implies that for all $\alpha \geq \alpha_0$, the equilibrium pricing policy coincides with that in equilibrium in the model without advertising. ■

Lemma 3. If $\hat{\pi} - (v - k) \geq 0$, then $F(p(\alpha))(k - v) + f(p(\alpha))(p(\alpha) - k)(p(\alpha) - v) \geq 0$ for all $p > \hat{p}$.

Proof of Lemma 3

Let $h(p) = F(p)(k - v) + f(p)(p - k)(p - v)$. At $p = \hat{p}$, we have

$$\begin{aligned} h(\hat{p}) &= F(\hat{p})(k - v) + f(\hat{p})(\hat{p} - k)(\hat{p} - v) = F(\hat{p})(k - v) + (1 - F(\hat{p}))(\hat{p} - v) \\ &= (1 - F(\hat{p}))(\hat{p} - k) - (v - k) \geq 0 , \end{aligned}$$

where the first equality follows because at $p = \hat{p}$ we have $1 - F(\hat{p}) - f(\hat{p})(\hat{p} - k) = 0$. From the concavity of $(1 - F(p))(p - k)$ it follows that function h is increasing:

$$h'(p) = (2f(p) + f'(p)(p - k))(p - v) > 0 .$$

Hence, $h(p) > 0$ for all $p > \hat{p}$. ■

Proof of Proposition 1

Part 1. To show that in equilibrium $\alpha_0 < \hat{\alpha}$, we argue by contradiction. Suppose, to the contrary, that $\alpha_0 \geq \hat{\alpha}$. By Lemmas 1 and 2, $V(\alpha)$ is strictly decreasing for all $\alpha_0 \leq \alpha < 1$. Therefore, (6) and (7)

hold with a strict inequality at $x(\alpha) = 0$ for all $\alpha_0 \leq \alpha < 1$. But then, from the continuity of the spot price in a mass market and (10), it follows that $x(\alpha) = 0$ is optimal as $\alpha \uparrow \alpha_0$. This yields the desired contradiction.

Furthermore, α_0 is unique. To see why, suppose that there are two equilibria: one with $\alpha_0 = \beta$ and another one with $\alpha_0 = \tilde{\beta}$, where $\beta < \tilde{\beta}$. By (6), in equilibrium with $\alpha_0 = \tilde{\beta}$, we have

$$\frac{\partial \pi(\alpha, p(\alpha))}{\partial p} (1 - \gamma) \frac{p(\alpha) - v}{\lambda} - (1 - \alpha) \frac{dV(\alpha)}{d\alpha} > 0 \text{ as } \alpha \uparrow \tilde{\beta}.$$

But this contradicts the optimality of no advertising as $\alpha \uparrow \tilde{\beta}$ in equilibrium with $\alpha_0 = \beta$ because subgame perfection implies that the price paths in these two equilibria must coincide for all $\alpha \geq \tilde{\beta}$.

Part 2. Consider the advertising rate $x(\alpha; \gamma)$ and the stopping point of the advertising policy $\alpha_0(\gamma)$ as functions of γ . Note that, from part 1 and the optimality condition for the switching point of the price policy in (10), it follows that $\partial V(\hat{\alpha}) / \partial \gamma = 0$ and $\partial p(\hat{\alpha}) / \partial \gamma = 0$, and hence, $\partial \hat{\alpha} / \partial \gamma = 0$ for any γ .

For all $\alpha \leq \hat{\alpha}$ the optimality condition (6) can be written more explicitly as

$$(1 - \alpha F(p(\alpha)) - \alpha f(p(\alpha))(p(\alpha) - k)) \frac{(1 - \gamma)(p(\alpha) - v)}{\lambda + (1 - \gamma)x(\alpha)} + (1 - \alpha) \frac{dV(\alpha)}{d\alpha} \leq c'(x(\alpha)). \quad (\text{A1})$$

Multiplying both sides by the rate of learning $\lambda + x(\alpha)$ and substituting into (A1) from (4) yields

$$\begin{aligned} & (1 - \alpha F(p(\alpha)))(k - \frac{(1 - \gamma)(\lambda + x(\alpha))}{\lambda + (1 - \gamma)x(\alpha)} v - \frac{\gamma \lambda}{\lambda + (1 - \gamma)x(\alpha)} p(\alpha)) \\ & - \alpha f(p(\alpha))(p(\alpha) - k)(p(\alpha) - v) \frac{(1 - \gamma)(\lambda + x(\alpha))}{\lambda + (1 - \gamma)x(\alpha)} + rV(\alpha) \leq c'(x)(x + \lambda) - c(x). \end{aligned} \quad (\text{A2})$$

Given a fixed $\tilde{\gamma}$, by (A2), we have the following for all $\alpha_0(\tilde{\gamma}) \leq \alpha < \hat{\alpha}$:

$$(1 - \alpha F(p(\alpha))(k - (\tilde{\gamma} p(\alpha) + (1 - \tilde{\gamma})v)) - \alpha(1 - \tilde{\gamma})f(p(\alpha))(p(\alpha) - k)(p(\alpha) - v) + rV(\alpha) \leq 0. \quad (\text{A3})$$

Differentiating the left-hand side of (A3) with respect to $\tilde{\gamma}$ and evaluating at $\alpha \uparrow \hat{\alpha}$ yields

$$-(1 - \alpha F(p(\alpha)) - \alpha f(p(\alpha) - k))(p(\alpha) - v) = -\frac{\partial \pi(\alpha, p(\alpha))}{\partial p} (p(\alpha) - v) < 0,$$

where the inequality follows because $\frac{\partial \pi(\alpha, p(\alpha))}{\partial p} > 0$ and $p(\alpha) - v > 0$ for all $\alpha_0 \leq \alpha < \hat{\alpha}$. Because

the left-hand side of (A3) is decreasing in $\tilde{\gamma}$, it must be that $x(\alpha) = 0$, $\partial V(\alpha) / \partial \gamma = 0$, and $\partial p(\alpha) / \partial \gamma = 0$ as $\alpha \uparrow \hat{\alpha}$ for all $\gamma > \tilde{\gamma}$. Stepping backwards and using the same arguments, it follows that this holds for all $\alpha \geq \alpha_0(\tilde{\gamma})$. This verifies that $\alpha_0(\gamma)$ is strictly decreasing in γ if $\alpha_0(\gamma) > 0$.

Part 3. We need to show that $\alpha_0(\gamma) = 0$ for γ close to 1. To obtain a contradiction, suppose that $\gamma = 1$ and $\alpha_0(1) > 0$. Then, by (A2), we should have

$$-(1 - \alpha_0(1))F(p(\alpha_0(1)))(p(\alpha_0(1)) - k) + rV(\alpha_0(1)) > 0 \text{ as } \alpha \uparrow \alpha_0(1).$$

But this is a contradiction because, by Lemmas 1 and 2, the flow profits are decreasing over time for all $\alpha > \alpha_0(1)$. This verifies part 3.

Part 4. We need to show that it must be that $\alpha_0(0) > 0$ if $\hat{\pi} \geq v - k$. To obtain a contradiction, suppose that $\alpha_0(\gamma) = 0$ for all γ . Then, by (A3), at $\alpha = 0$ it must be that $k - v + rV(0) \leq 0$, or

$$\hat{\pi} < rV(0) \leq v - k,$$

where the first inequality follows by Lemmas 1 and 2. This yields the desired contradiction. Therefore, it must be that for all sufficiently small γ we have $x(\alpha) > 0$ for some $\alpha < \hat{\alpha}$. ■

Proof of Proposition 2

If $\hat{w} < \hat{p}$, there must exist $\hat{\alpha} < 1$ such that for all $\alpha > \hat{\alpha}$ the marginal buyer is an informed buyer because (11) must be less than (12) as $\alpha \rightarrow 1$; that is, sacrificing the current profits in order to let buyers learn by purchasing cannot be optimal once the number of uninformed buyers is sufficiently small. Therefore, for all $\alpha(t) \geq \hat{\alpha}$, the seller's value function satisfies equation (5), or more explicitly

$$V(\alpha(t)) = \int_t^\infty e^{-r(s-t)} [(1 - (1 - \alpha(t))e^{-\int_t^s \gamma x(\alpha(z)) dz}) \hat{\pi} - c(x(\alpha(s)))] ds$$

with

$$\frac{dV(\alpha(t))}{d\alpha(t)} = \hat{\pi} \int_t^\infty e^{-\int_t^s (r + \gamma x(\alpha(z)) dz) ds} \in (0, \hat{\pi} / r], \quad (\text{A4})$$

and

$$\gamma x(\alpha) \frac{d[V'(\alpha)(1 - \alpha)]}{d\alpha} = rV'(\alpha) - \hat{\pi} \leq 0 \text{ for all } \alpha > \hat{\alpha}. \quad (\text{A5})$$

The last inequality follows by (A4).

Part 1. To show that $\alpha_0 = \hat{\alpha}$ if $\gamma = 0$, we first show that $x(\alpha) = 0$ for all $\alpha > \hat{\alpha}$, and then we show that $\lim_{\alpha \uparrow \hat{\alpha}} x(\alpha) > 0$. By assumption, the marginal buyer is an informed buyer for all $\alpha > \hat{\alpha}$, and hence, from (8) it immediately follows that $x(\alpha) = 0$ for all $\alpha > \hat{\alpha}$ when $\gamma = 0$. By (6), we have

$$c'(x(\alpha)) = \frac{\partial \pi(\alpha, p(\alpha))}{\partial p} \frac{\partial p(\alpha)}{\partial x(\alpha)} + \frac{dV(\alpha)}{d\alpha} (1 - \alpha) > 0 \text{ as } \alpha \uparrow \hat{\alpha},$$

where $\lim_{\alpha \uparrow \hat{\alpha}} \frac{\partial p(\alpha)}{\partial x(\alpha)} = E[V^\theta(\hat{\alpha})] - V^u(\hat{\alpha}) = \int_i^\infty e^{-\int_i^t (r+x(\alpha(s)))ds} E \max[\theta - \hat{p}, 0] dt$. The inequality follows because $p(\alpha) < \hat{p}$ and $\frac{\partial \pi(\alpha, p(\alpha))}{\partial p} > 0$ as $\alpha \uparrow \hat{\alpha}$, and by (A4), $dV(\hat{\alpha})/d\alpha > 0$. So it must be that $\alpha_0 = \hat{\alpha}$.

Part 2. If $\gamma > 0$, from (8) and (A4), it immediately follows that $x(\alpha) > 0$ for all $\hat{\alpha} \leq \alpha < 1$. Hence, it must be that $\alpha_0 = 1$. ■

Proof of Proposition 3

From Lemma 2 and Proposition 1 it follows that $0 < \alpha_0 < \hat{\alpha} = \hat{\alpha}_n$ and $p(\hat{\alpha}) = p(\hat{\alpha}_n)$. By Lemma 1, we also have

$$\lim_{r \rightarrow 0} p(\hat{\alpha}) = v + \lambda^2 \int_i^\infty \int_i^T e^{-r(s-i)} E \max[p^m(\alpha(t)) - \theta, 0] dt \lambda e^{-\lambda(T-i)} dT = \theta_h.$$

This can be verified by observing that $p^m(\alpha)$ is independent of r for all $\alpha > \hat{\alpha}$. This implies that, all else equal, $p(\hat{\alpha})$ must increase as r decreases. But then from (10) it follows that $\hat{\alpha}$ must decrease because $p(\hat{\alpha}) = p_m(\hat{\alpha})$ and $dp^m/d\alpha < 0$. Hence, as $r \rightarrow 0$, we have $\hat{\alpha} \rightarrow 1/(1 + f(\theta_h)(\theta_h - k))$ and $p(\hat{\alpha}) \rightarrow \theta_h$.

To show that $x'(\alpha) < 0$ for all $0 \leq \alpha < \alpha_0$, we argue by contradiction. Suppose that $x'(\alpha) \geq 0$ for some $\alpha \in (0, \alpha_0)$ as $r \rightarrow 0$. Because $x(\alpha) = 0$ for all $\alpha \geq \alpha_0$, continuity of $x(\alpha)$ in a mass market implies that $x'(\alpha) < 0$ as $\alpha \uparrow \alpha_0$. So consider the largest $\alpha_1 < \alpha_0$ such that $x'(\alpha_1) = 0$.

From Lemma 1 it also follows that $p(\alpha)$ is decreasing for all $\alpha \geq \alpha_0$; that is,

$$\frac{dp}{d\alpha} \frac{d\alpha}{dt} = r(p(\alpha) - v) - \lambda E \max[\theta - p(\alpha), 0] < 0 \text{ for all } \alpha \geq \alpha_0.$$

Therefore, by (9), it follows that $p(\alpha_1) > p(\alpha_0)$ because, by assumption, $x'(\alpha) < 0$ for all $\alpha_1 < \alpha < \alpha_0$. Hence, the value of information changes very little because the spot prices are high as $r \rightarrow 0$; that is,

$$\begin{aligned} \lim_{r \rightarrow 0} \frac{d(EV^\theta(\alpha_1) - V^u(\alpha_1))}{d\alpha} (\lambda + x(\alpha_1))(1 - \alpha_1) \\ = \lim_{r \rightarrow 0} r(p(\alpha_1) - v) - (\lambda + x(\alpha_1))E \max[\theta - p(\alpha_1), 0] = 0, \end{aligned}$$

because, by Lemmas 1 and 2, $p(\alpha_0) > p(\hat{\alpha})$ and $p(\hat{\alpha}) \rightarrow \theta_h$ as $r \rightarrow 0$.

Now we are ready to show that the advertising rate must be decreasing for a sufficiently small discount rate. At $\gamma = 0$ and $x(\alpha) > 0$, the equation (A2) becomes

$$\begin{aligned} (1 - \alpha F(p(\alpha)))(k - v) - \alpha f(p(\alpha))(p(\alpha) - k)(p(\alpha) - v) + rV(\alpha) \\ = c'(x(\alpha))(x + \lambda) - c(x(\alpha)). \end{aligned} \tag{A6}$$

Differentiating both sides of (A6) with respect to α and evaluating at $\alpha = \alpha_1$ with $dx(\alpha_1)/d\alpha = 0$ yields:

$$\begin{aligned} & -\alpha_1[2f(p(\alpha_1)) + f'(p(\alpha_1))(p(\alpha_1) - k)] \frac{p(\alpha_1) - v}{1 - \alpha_1} (r \frac{p(\alpha_1) - v}{\lambda + x(\alpha_1)} - E \max[\theta - p(\alpha_1), 0]) \\ & + rV'(\alpha_1) - [F(p(\alpha_1))(k - v) + f(p(\alpha_1))(p(\alpha_1) - k)(p(\alpha_1) - v)] = 0. \end{aligned}$$

However, note that the first and second terms vanish as $r \rightarrow 0$, while the third term on the right-hand side is strictly negative by Lemma 3 because $p(\alpha_1) > p(\alpha_0) > p(\hat{\alpha}) \geq \hat{p}$. This yields the desired contradiction. So it must be that $dx(\alpha_1)/d\alpha < 0$ if $dx(\alpha)/d\alpha < 0$ for all $\alpha_1 < \alpha < \alpha_0$, and hence, it must be that $dx(\alpha)/d\alpha < 0$ for all $\alpha < \alpha_0$ when r is sufficiently small. ■

Proof of Proposition 4

For $\gamma = 1$ the differential equations that govern the evolution of the equilibrium spot price (9) and the advertising rate (13) for all $\alpha < \hat{\alpha}$ become, respectively,

$$\frac{dp}{d\alpha} \frac{d\alpha}{dt} = (r + x(\alpha))(p(\alpha) - v) - \lambda E \max[\theta - p(\alpha), 0] \text{ and} \quad (\text{A7})$$

$$c''(x(\alpha)) \frac{dx(\alpha)}{d\alpha} \frac{d\alpha}{dt} = rc'(x(\alpha)) - (1 - \alpha) \frac{d\pi(\alpha, p(\alpha))}{d\alpha}. \quad (\text{A8})$$

Also, it will be useful to let $\bar{w}(x)$ denote the rest point of (A7) for a given constant advertising rate x , which uniquely solves

$$(r + x)(\bar{w}(x) - v) - \lambda E \max[\theta - \bar{w}(x), 0] = 0$$

By rearranging this equation it can be seen that $\bar{w}(x)$ is the price that extracts full surplus from the uninformed buyers if the seller charges that price and advertises at rate x in the future. Note that $\hat{w} < \bar{w}(0)$ because $\hat{w} < \hat{p}$, and $\bar{w}(x)$ is decreasing in x .

The proof proceeds in six steps. In Step 1, we show that for all $\alpha < \hat{\alpha}$ the flow profit is decreasing in α if the price $p(\alpha) \leq p^m(\alpha)$ is decreasing. In Steps 2 and 3, we show that $p(\alpha)$ is decreasing as $\alpha \uparrow \hat{\alpha}$ as well as for all $\alpha < \hat{\alpha}$, provided that x_h is sufficiently small. In Step 4, we show that $x(\alpha)$ is nondecreasing for all $\alpha < \hat{\alpha}$ and that the start of advertising may be delayed. In Step 5, we show that $x(\alpha)$ is strictly decreasing for all $\hat{\alpha} < \alpha < 1$, and we verify the uniqueness of $\hat{\alpha}$ in Step 6.

Step 1. Note that $\pi(\alpha, p(\alpha))$ is decreasing in α if the price $p(\alpha)$ is nonincreasing and $p(\alpha) \leq p^m(\alpha)$ because differentiating $\pi(\alpha, p(\alpha))$ yields

$$\frac{d\pi(\alpha, p(\alpha))}{d\alpha} = \frac{\partial \pi(\alpha, p(\alpha))}{\partial \alpha} + \frac{\partial \pi(\alpha, p(\alpha))}{\partial p} \frac{dp(\alpha)}{d\alpha} < 0. \quad (\text{A9})$$

The inequality follows because $\partial \pi(\alpha, p(\alpha))/\partial \alpha = -F(p(\alpha))(p(\alpha) - k) < -F(p(\alpha))(v - k) \leq 0$, and concavity of $(1 - F(p))(p - k)$ implies that $\partial \pi(\alpha, p(\alpha))/\partial p \geq 0$ when $p(\alpha) \leq p^m(\alpha)$.

Step 2. Next we show that $\lim_{\alpha \uparrow \hat{\alpha}} dp(\alpha)/d\alpha < 0$ for all $x_h > 0$ that are sufficiently small, specifically if $\hat{w} \leq \bar{w}(x_h)$. By the assumption that the marginal buyer is an informed buyer for all $\alpha > \hat{\alpha}$, at $\alpha = \hat{\alpha}$ the uninformed buyers are willing to pay

$$\lim_{\alpha \uparrow \hat{\alpha}} p(\alpha) = v + \lambda \int_i^\infty e^{-\int_i^t (r+x(\alpha(s)))ds} E \max[\theta - \hat{p}, 0] dt \leq \hat{w} \leq \bar{w}(x_h) < \bar{w}(x(\hat{\alpha})).$$

The second inequality follows by the maintained assumption that $\hat{w} \leq \bar{w}(x_h)$. The third inequality follows because $d\bar{w}(x)/dx < 0$ and $x(\hat{\alpha}) < x_h$.

Step 3. Next we show that in equilibrium,

$$p(\alpha) \leq \bar{w}(x(\alpha)) \text{ for all } \alpha \leq \hat{\alpha}. \quad (\text{A10})$$

For a fixed advertising rate $x(\alpha) = x$, the argument is rather straightforward and can proceed as follows. Since $\bar{w}(x)$ is the level of the price that makes the right-hand side of equation (A7) zero, and the right-hand side of this equation is increasing in $p(\alpha)$, the fact that $p(\alpha) < \bar{w}(x)$ as $\alpha \uparrow \hat{\alpha}$ implies that at the equilibrium price, the right-hand side of equation (A7) is negative, which in turn implies that $p(\alpha)$ must be decreasing for all $\alpha < \hat{\alpha}$. Next we show that this is also true when equilibrium $x(\alpha)$ is a function of α .

The following argument is somewhat more complicated because the equilibrium advertising rate and price are determined jointly. We argue by contradiction. Suppose, to the contrary, that $p(\alpha) > \bar{w}(x(\alpha))$ for some $\alpha \in [0, \hat{\alpha})$. Then because, by Step 2, $\lim_{\alpha \uparrow \hat{\alpha}} p(\alpha) < \bar{w}(x(\hat{\alpha}))$, continuity of $p(\alpha)$ and $\bar{w}(x(\alpha))$ implies that there exists $\tilde{\alpha} \in (0, \hat{\alpha})$ and $\varepsilon > 0$ such that $p(\alpha) > \bar{w}(x(\alpha))$ for all $\alpha \in (\tilde{\alpha} - \varepsilon, \tilde{\alpha})$ and $p(\tilde{\alpha}) = \bar{w}(x(\tilde{\alpha}))$. Then from (A8) and (A9) it follows that $\frac{dx(\tilde{\alpha})}{d\alpha} \geq 0$ since, by assumption, $\frac{dp(\tilde{\alpha})}{d\alpha} = 0$, and hence, $\frac{d\bar{w}(x(\tilde{\alpha}))}{dx(\tilde{\alpha})} \frac{dx(\tilde{\alpha})}{d\alpha} \leq 0$. But this implies that $p(\alpha) \leq \bar{w}(x(\alpha))$ for some $\alpha \in (\tilde{\alpha} - \varepsilon, \tilde{\alpha})$, which gives us the desired contradiction. Hence, it must be that in equilibrium $p(\alpha) \leq \bar{w}(x(\alpha))$ and $p(\alpha)$ is strictly decreasing for all $\alpha < \hat{\alpha}$.

Step 4. Because, as we just showed, $p(\alpha)$ is decreasing for all $\alpha < \hat{\alpha}$, by (A9), $\pi(\alpha, p(\alpha))$ is decreasing in α for all $\alpha < \hat{\alpha}$ since $p(\alpha) \leq p^m(\alpha)$. Hence, from (A8) and the strict convexity of the advertising cost function it follows that $x(\alpha)$ is strictly increasing if $x(\alpha) > 0$ for $\alpha < \hat{\alpha}$. Also note that, by (6) and (A4), we have $\lim_{\alpha \uparrow \hat{\alpha}} x(\alpha) > 0$. Hence, it must be that either (i) there exists $\alpha^* \in [0, \hat{\alpha})$ such that $x(\alpha) = 0$ for all $\alpha \leq \alpha^*$, and $x(\alpha) > 0$ if $\alpha > \alpha^*$, or (ii) $x(\alpha) > 0$ for all $\alpha \geq 0$ and $\alpha^* = 0$.

Next we show that in equilibrium $\alpha^* > 0$ if $v - k \geq \hat{\pi}$. Suppose that, to the contrary, $\alpha^* = 0$. Then, by (6), it must be that

$$V'(0) = c'(x(0)) \geq 0, \quad (\text{A11})$$

and we have

$$\begin{aligned} c'(x(0))x(0) - c(x(0)) &< c'(x(0))(\lambda + x(0)) - c(x(0)) = V'(0)(\lambda + x(0)) - c(x(0)) \\ &= rV(0) - \pi(0, p(0)) = r \left(\int_0^{\hat{\alpha}} e^{-rs} (\pi(\alpha(s), p(\alpha(s))) - c(x(\alpha(s)))) ds \right) \end{aligned}$$

$$+ \int_t^\infty e^{-rs} (\alpha(s)\hat{\pi} - c(x(\alpha(s))))ds - \pi(0, p(0)) \leq r \int_0^\infty e^{-rs} \pi(0, p(0))ds - \pi(0, p(0)) = 0.$$

The first equality follows from (A11). The second equality follows by rearranging (4) evaluated at $\alpha = 0$. The second inequality follows because, by (A9), $\pi(0, p(0)) > \pi(\alpha, p(\alpha))$ for all $\alpha \leq \hat{\alpha}$ and, by assumption, $\pi(0, p(0)) = p(0) - k \geq v - k \geq \hat{\pi}$. But this yields the desired contradiction because the convexity of $c(x)$ implies that $xc'(x) > c(x)$ for all $x > 0$. Therefore, there exists $\alpha^* \in (0, \hat{\alpha})$ such that $V'(\alpha) < 0$ and $x(\alpha) = 0$ for all $0 \leq \alpha < \alpha^*$.

Step 5. For all $\alpha > \hat{\alpha}$, equilibrium advertising rate $x(\alpha)$ satisfies the following differential equation

$$c''(x(\alpha)) \frac{dx}{d\alpha} \frac{d\alpha}{dt} = rc'(x(\alpha)) - (1 - \alpha)\hat{\pi} = (1 - \alpha)(r \frac{dV(\alpha)}{d\alpha} - \hat{\pi}) < 0,$$

where the inequality follows by (A5). Hence, from the strict convexity of the advertising cost function it follows that $x(\alpha)$ is decreasing for all $\alpha > \hat{\alpha}$.

Step 6. From (6) and (8) it follows that the advertising path is continuous as $\lim_{\alpha \uparrow \hat{\alpha}} x(\alpha)$

$= \lim_{\alpha \downarrow \hat{\alpha}} x(\alpha) = x(\hat{\alpha})$ for $\gamma = 1$. So, by (11) and (12), a stopping point $\hat{\alpha} \in [0, 1)$ solves

$$g(\hat{\alpha}) \equiv \pi(\hat{\alpha}, v + \lambda(EV^\theta(\hat{\alpha}) - V^u(\hat{\alpha}))) + \lambda(1 - \hat{\alpha}) \frac{dV(\hat{\alpha})}{d\alpha} - \hat{\alpha}(1 - F(\hat{p}))(\hat{p} - k) \leq 0,$$

where $EV^\theta(\hat{\alpha}) - V^u(\hat{\alpha}) = \int_t^\infty e^{-\int_t^s (r+x(\alpha(s)))ds} E \max[\theta - \hat{p}, 0] dt$. Differentiating g for any $\alpha(t) \geq \hat{\alpha}$

yields

$$\begin{aligned} \frac{dg(\alpha)}{d\alpha} &\equiv \frac{\partial \pi(\alpha, p(\alpha))}{\partial \alpha} + \lambda \frac{d((1 - \alpha)dV(\alpha)/d\alpha)}{d\alpha} - \hat{\pi} \\ &+ \frac{\partial \pi(\alpha, p(\alpha))}{\partial p} \lambda \int_t^\infty e^{-\int_t^s (r+x(\alpha(s)))ds} (x(\alpha(t)) - x(\alpha(t + \tau))) E \max[\theta - \hat{p}, 0] d\tau < 0 \end{aligned}$$

because $x_h \rightarrow 0$. The inequality follows because the first line is strictly negative since

$\partial \pi(\alpha, p(\alpha))/\partial \alpha < 0$ for all $\alpha < 1$, and by Step 5, $(1 - \alpha)dV(\alpha)/d\alpha = c'(x(\alpha))$ is strictly decreasing for all $\hat{\alpha} < \alpha < 1$; and the expression in the second line goes to 0 as $x(\alpha) < x_h \rightarrow 0$.

Therefore, perfection implies that function $g(\alpha)$ is strictly decreasing for all $\alpha \geq \hat{\alpha}$, which establishes the uniqueness of $\hat{\alpha}$. ■

Proof of Proposition 5

Part 1. This follows immediately by Lemma 2 and Proposition 1.

Part 2. First consider the case with $\gamma = 0$. To show that $\hat{\alpha} > \hat{\alpha}_n$, we suppose, to the contrary, that $\hat{\alpha} \leq \hat{\alpha}_n$. Then we have

$$\begin{aligned} 0 \leq g_n(\hat{\alpha}) &= \pi(\hat{\alpha}, \hat{w}) + \frac{\hat{\pi}}{r} \lambda(1 - \hat{\alpha}) - \hat{\alpha}\hat{\pi} \\ &< \lim_{\alpha \uparrow \hat{\alpha}} \pi(\alpha, p(\alpha)) - c(x(\alpha)) + \frac{\hat{\pi}}{r} (\lambda + x(\alpha))(1 - \alpha) - \alpha\hat{\pi}. \end{aligned}$$

The first inequality follows because $g_n(\alpha)$ is decreasing in α . The second inequality follows because the seller can always set $x(\alpha) = 0$ as $\alpha \uparrow \hat{\alpha}$, but a higher profit can be achieved by advertising. Therefore, at $\alpha = \hat{\alpha}$ the payoff in (11) is greater than in (12), which yields the desired contradiction.

Now we suppose that $\gamma = 1$. To show that $\hat{\alpha} < \hat{\alpha}_n$, suppose, to the contrary, that $\hat{\alpha} \geq \hat{\alpha}_n$. Then we have

$$0 \geq g_n(\hat{\alpha}) = \pi(\hat{\alpha}, \hat{w}) + \frac{\hat{\pi}}{r} \lambda(1 - \hat{\alpha}) - \hat{\alpha} \hat{\pi} > \lim_{\alpha \uparrow \hat{\alpha}} \pi(\alpha, p(\alpha)) + \frac{dV(\alpha)}{d\alpha} \lambda(1 - \alpha) - \alpha \hat{\pi}.$$

The first inequality follows because $g_n(\alpha)$ is decreasing in α . The second inequality follows because $\lim_{\alpha \uparrow \hat{\alpha}} p(\alpha) = v + \lambda \int_i^\infty e^{-\int_i^t (r+x(\alpha(s)))ds} E \max[\theta - \hat{p}, 0] dt < \hat{w}$ and by (A4). Therefore, either at $\hat{\alpha} = 0 \geq \hat{\alpha}_n$ or at $\alpha = \hat{\alpha} > 0$, the payoff in (11) is less than in (12). This yields the desired contradiction.

Part 3. In a mass market, this must be true because the uninformed buyers never stop learning by purchasing, and so $\alpha(t) > \alpha_n(t)$ for all $t > \tilde{t} = \inf\{t : x(\alpha(t)) > 0\}$. In a niche market, this must be true because, by Proposition 2, part 2 of this proposition, and Lemma 1, we have $\lim_{t \rightarrow \infty} \alpha(t) = \alpha_0 > \hat{\alpha}_n = \lim_{t \rightarrow \infty} \alpha_n(t)$. ■

Proof of Proposition 6

Part 1. Suppose that $\gamma = 0$. For $\hat{w} \geq \hat{p}$, we have

$$V_n^u(0) = e^{-r\hat{t}_n} V_n^u(\hat{\alpha}_n) < e^{-r\hat{t}} V_n^u(\hat{\alpha}_n) = e^{-r\hat{t}} V^u(\hat{\alpha}) = V^u(0).$$

The first and last equalities follow because the uninformed buyers obtain zero net flow payoff during the early stages. The inequality follows because in a mass market $\hat{\alpha} = \hat{\alpha}_n$, which implies that $\hat{t} < \hat{t}_n$ because the size of the informed segment grows faster when the seller advertises. The second equality follows by Lemma 2 and Proposition 5. For $\hat{w} < \hat{p}$, we have

$$V_n^u(\alpha_n(t)) = V^u(\alpha(t)) = 0 \text{ for all } t,$$

because in a niche market the uninformed obtain zero flow utility forever once the price is raised to \hat{p} , and they are left indifferent between purchasing and delaying purchasing before that.

Now suppose that $\gamma > 0$. Again we start with the case that $\hat{w} < \hat{p}$. Now $V^u(0) > V_n^u(0) = 0$ because the uninformed buyers obtain flow of valuable information $\gamma x(\alpha)(EV^\theta(\alpha) - V^u(\alpha)) > 0$ for all α such that $x(\alpha) > 0$. Similarly, for $\hat{w} \geq \hat{p}$, advertising strictly increases the discounted sum of payoffs for the uninformed buyers as $\hat{t} < \hat{t}_n$ and $V_n^u(\hat{\alpha}_n) = V^u(\hat{\alpha})$ for all γ .

Part 2. Suppose that $\hat{w} \geq \hat{p}$. By Lemma 1, $p_n(\alpha)$ is decreasing for all α , and it must be that $\alpha(t) > \alpha_n(t)$ as $t \uparrow t_0 < \hat{t} < \hat{t}_n$ and for all $t \geq t_0$. Therefore, by Lemma 2, we have $p(\alpha(t)) = p_n(\alpha(t)) < p_n(\alpha_n(t))$ for all $\alpha(t) \geq \alpha_0$. So from continuity of the price path, it follows that there exists $\tilde{t} < t_0$ such that for all $t \geq \tilde{t}$ we have

$$p(\alpha(t)) < p_n(\alpha_n(t)),$$

where $\alpha(t_0) = \alpha_0$. This implies that consumers are necessarily better off for all $t \geq \tilde{t}$ in equilibrium with advertising because more of them are informed and they pay lower prices.

Now we suppose that $\hat{w} < \hat{p}$. By Proposition 5, there exists $\bar{t} < \infty$ such that we have

$$CS(t) = \int_t^\infty e^{-r(s-t)} \alpha(s) E \max[\theta - \hat{p}, 0] ds \geq \int_t^\infty e^{-r(s-t)} \alpha_n(s) E \max[\theta - \hat{p}, 0] ds = CS_n(t)$$

for all $t \geq \bar{t}$. ■

Proof of Proposition 7

First observe that it must be that $V(\alpha) > V_n(\alpha)$ as $\alpha \uparrow \alpha_0$ because, by Lemma 2, $EV^\theta(\alpha) - V^u(\alpha)$, the continuation profits are the same in equilibrium with and without advertising for all $\alpha \geq \alpha_0$, and the monopolist achieves a strictly higher payoff by choosing $x(\alpha) > 0$ as $\alpha \uparrow \alpha_0$. But, by Proposition 1, we can pick $\gamma \in [0, 1)$ such that $\alpha_0 > 0$ is arbitrarily close to 0. Hence, it must be that $V(\alpha) \geq V_n(\alpha)$ for all $0 \leq \alpha < \alpha_0 \downarrow 0$, and by Proposition 6, we have $W(0) \geq W_n(0)$ for some $\gamma < 1$. ■

Proof of Proposition 8

In a mass market, the flow social welfare is given by

$$\omega(\alpha(t), p(\alpha(t))) \equiv \alpha(t) \int_{p(\alpha(t))}^{\theta_h} (\theta - k) dF(\theta) + (1 - \alpha(t))(v - k).$$

Note that $\partial \omega(\alpha, p(\alpha)) / \partial p < 0$ and $\partial \omega(\alpha, p(\alpha)) / \partial \alpha = \int_{\theta_l}^{p(\alpha)} (k - \theta) dF(\theta) > 0$ for $p(\alpha)$ close to \hat{p} when $k \geq E[\theta | \theta \leq \hat{p}]$. But, as shown in the proof of part 2 of Proposition 6, there exists $\tilde{t} < t_0$ such that for all $t \geq \tilde{t}$ we have $p(\alpha(t)) < p_n(\alpha_n(t))$ and $\alpha(t) > \alpha_n(t)$. Therefore, as $p(\alpha(t)) \rightarrow \hat{p}$ for $t \rightarrow \infty$, it must be that for sufficiently large $t < \infty$ we have $\omega(\alpha(t), p(\alpha(t))) \geq \omega(\alpha_n(t), p_n(\alpha_n(t)))$, and hence,

$$W(t) = \int_t^\infty e^{-r(s-t)} \omega(\alpha(s), p(\alpha(s))) ds \geq \int_t^\infty e^{-r(s-t)} \omega(\alpha_n(s), p_n(\alpha_n(s))) ds = W_n(t).$$

In a niche market, by Proposition 5, for sufficiently large t we have

$$W(t) = \int_t^\infty e^{-r(s-t)} [\alpha(s) \int_{\hat{p}}^{\theta_h} (\theta - k) dF(\theta) - c(x(\alpha(s)))] ds \geq \frac{1}{r} \hat{\alpha}_n \int_{\hat{p}}^{\theta_h} (\theta - k) dF(\theta) ds = W_n(t)$$

because $\lim_{t \rightarrow \infty} W(t) = \frac{1}{r} \alpha_0 \int_{\hat{p}}^{\theta_h} (\theta - k) dF(\theta) > \frac{1}{r} \hat{\alpha}_n \int_{\hat{p}}^{\theta_h} (\theta - k) dF(\theta) = \lim_{t \rightarrow \infty} W_n(t)$. ■

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